JAST, Vol. 6, No. 2, pp 79–85
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JAST Journal of Aerospace Science and Technology

Low-Thrust Optimal Orbit Raising with Plane Change

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A new guidance scheme for the problem of Low-thrust transfer between inclined orbits is developed within the framework of optimal control theory. The objective of the guidance scheme is to provide the appropriate thrust steering program that will transfer the vehicle from an inclined low earth orbits to the high earth orbits. The presented guidance scheme is determined, using optimal control theory such that minimum time performance measure is determined and boundary conditions for these unspecified final time problems are satisfied. One of the novelties of this work is changing independent variable from time to thrust angle and considering properties of autonomous system equations to reduce to one where exact analytical solution is obtained.

INTRODUCTION

Optimal low-thrust orbit transfer has received a great deal of attention in the astrodynamics and flight mechanics literature over the past decades. The evolution of low-thrust propulsion technologies has reached a point where such systems have become an economical option for many space mission applications. Also the development of efficient control laws has received an increasing amount of attention in recent years, and few studies have examined the subject of inclination changing maneuvers [1-3].

Many applications of this problem involve low-thrust propulsion systems where the orbit transfer takes place over a relatively long duration. For example, the minimum-time transfer of a 100 kg spacecraft from low Earth (LEO) to geostationary orbit (GEO) using a 1-N thruster take about 5 days, as compared with about 5 h for a Hohman transfer, but the continuous thrust case would typically use much less propellant. The thrust-angle profile of a constant –thrust orbit transfer depends on both the thrust magnitude and the size of the orbit transfer [4].

Analytical solutions of the low-thrust problem are very useful in preliminary mission analyses as well as spacecraft system design and optimization. The overall design of a solar electric transfer vehicles or even an integrated spacecraft requires extensive parametric analyses for optimum sizing of the various power, propulsion, and thermal management systems to maximize delivered payload to the destination orbit. These parametric studies require hundreds of iterations, precluding the use of the numerically generated transfer solutions. The analytic solutions are also desirable for future onboard autonomous guidance applications, especially for smaller spacecraft such as in the mini-and microsatellite category where the application of low-thrust technology for orbit maintenance and control is most efficient [5, 6].

Analytic expressions for the maximum change in inclination between two circular orbits of given size with the continuous constant acceleration and fixed transfer time is derived by Edelbum. Conversely, he derived an analytical expression for the total velocity needed to carry out the transfer between given inclined circular orbits. This theory was later generalized by Wiesel and Alfano [7], who allowed for the variation of the out-of-plane or thrust yaw angle during each revolution, unlike Edelbum, who used the simpler constant yaw profile. Thus, the semi major axis and inclination space was mapped by direct numerical integration of the simplified differential equations, such that the minimum time for a given transfer is read directly from the solution map. In Ref. [8] the optimal thrust pith and yaw profiles required for a given transfer were determined in a semi analytical way by also considering discontinues thrust due to eclipsing. In Ref. [9], rapid

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transfer calculations were demonstrated by analytic modeling of the various transfer parameters including shadowing and solar power degradation effects due to the Van Allen radiation belts. The thrust yaw angle is held constant through the transfer, and the required value is determined by iteration. This not as optimal as the Edelbum steering solution, which holds the yaw angle constant during each revolution but varies its value from revolution to revolution in an optimal manner. All of these analyses assume that the orbit remains or is forced to be circular after each cycle or revolution. In Ref. [5] the original Edelbum theory is revisited by first extending and completing it and by deriving the yaw steering expressions without ambiguity and recasting the theory within the framework of optimal control for minimum time. In Ref. [11], the author looks again at Edelbum's approach but some improvements are introduced, while maintaining the assumption of quasi-circular orbits. Trajectories with variable thrust and specific impulse at constant power are analyzed.

We begin by defining the idealized model and the equations of motion. The equations are considered as Edelbum's model. Minimum-time transfers is established, which needs to solve a tow-point boundary value problem requiring the determination of the unknown initial and final parameters for the Lagrange multipliers or costates. In the present paper, all solutions are obtained exact analytically and spacecraft is modeled point-mass that moving in inclined plane and being controlled with a constant thrust with variable direction.

OPTIMAL CONTROL IN DYNAMIC SYSTEMS

Optimal control problems of dynamics systems can be formulated using calculus of variations. In this regard, one usually assumes mathematical representation of the system tinder study as a first-order differential equation:

$$\dot{\vec{x}}(t) = f\left[\vec{x}(t), \vec{u}(t), t\right], \qquad t_0 \le t \le t_f \tag{1}$$

where $\vec{x}(t)$ denotes the n-states and $\vec{u}(t)$ is the vector of m-control components. The second step in formulation of optimal control problem is to introduce an appropriate performance function. A conventional form can be expressed as:

$$J = \phi\left[\vec{x}(t_f), t_f\right] + \int_{t_0}^{t_f} L\left[\vec{x}(t), \vec{u}(t), t\right] dt \tag{2}$$

where $\phi\left[\vec{x}(t_f), t_f\right]$ is the penalty function for the final states at the final time. Additionally, there are could exist terminal constrains in functional form for final time unspecified situations, presented as:

$$\phi\left[\vec{x}(t_f), t_f\right] = 0 \tag{3}$$

Subsequently, the governing differential equations as well as the terminal constraints are augmented to the performance function using Lagrange multipliers $\nu(t)$ and $\lambda(t)$, respectively.

$$J = \left[\phi + \nu^T \psi\right]_{t=t_f} + \int_{t_0}^{t_f} \left\{ L(x, \vec{u}, t) + \lambda^T \left[f(x, \vec{u}, t) - \dot{\vec{x}} \right] \right\} dt \tag{4}$$

Having officially formulated the problem, the first step toward a variational solution is to determine the system Hamiltonian

$$H = L(x, \vec{u}, t) + \lambda^T(t) f(x, \vec{u}, t) \tag{5}$$

Based on the Hamiltonian, the necessary condition for an optimal solution are:

I. the state equation:

$$\dot{x} = \frac{\partial H}{\partial \vec{\lambda}} = f\left[\vec{x}(t), \vec{u}(t), t\right] \tag{6}$$

II. the costate equation:

$$\dot{\lambda} = -\left(\frac{\partial H}{\partial \vec{x}}\right)^T = -\left(\frac{\partial f}{\partial \vec{u}}\right)^T \lambda - \left(\frac{\partial L}{\partial \vec{u}}\right)^T \tag{7}$$

III. and the optimality condition:

$$0 = \left(\frac{\partial H}{\partial \vec{u}}\right)^T = \left(\frac{\partial f}{\partial \vec{u}}\right)^T \lambda + \left(\frac{\partial L}{\partial \vec{u}}\right)^T \tag{8}$$

The above equations need to be simultaneously satisfied, considering an appropriate set of initial and boundary conditions given below;

IV. initial conditions:

$$\vec{x}_k(t_0) = (is\ known), \qquad \lambda_k(t_0) = 0 \tag{9}$$

V. orthogonality conditions:

$$\lambda(t_f) = \left(\frac{\partial \phi}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right)^T \tag{10}$$

$$\varphi = \left[\frac{\partial \phi}{\partial t} + \nu^T \frac{\partial \psi}{\partial t} + \left(\frac{\partial \phi}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right) f + L\right] \tag{11}$$

VI. terminal conditions.

$$\psi\left[x(t_f), t_f\right] = 0 \tag{12}$$

The optimality condition (8) usually allows for optimal determination of the m-control components as functions of the states and the costates. The solution of the 2n differential Eqs. (6) and (7) are to be considered with the aid of $2n + l + q$ boundary conditions specified in Eqs. (9)-(12). Since in most practical applications, the governing equations are non-linear, one dose not usually expert to obtain an analytically solution but it is very important to get analytical solution for problem [12-14].

LOW-THRUST ORBITAL TRANSFER

The problems are formulated as an optimal control problems with the thrust direction being the control variable. The advantage of these guidance laws are that, they are the exact solutions to the two-point boundary value problems that we can name them as hypersensitive optimal control problem which satisfied terminal conditions for time free changing planes maneuvers for low-thrust spacecrafts. Furthermore, in these methodologies, the several difficulties associated with the numerical determination of optimal control solutions for nonlinear systems, such as slow convergence rate and high sensitivity to problem are removed.

In this paper, the investigation is performed by solving optimal control problem for three performance indexes and using the Edelbaum's equation to describe the underlying dynamics. Edelbaum used Lagrange planetary equations of orbital motion to develop new, simply and applicable equations set. As mentioned, problems such that arise frequently in astrodynamics applications which often the control resources available for achieving desired objectives, so that finding an optimal control strategy analytically, or exact solution to satisfying performance measures are very important. There are important problems which play keys role in finding space vehicle trajectories based on the performance indexes such as time, effort, fuel, tracking errors and etc [6].

The spacecraft status and its conditions in this work are described as follow:

a) Assuming constant acceleration and yaw angle as control variable.

b) Assuming orbital inclination and velocity as the state variables.

c) Each revolution is considered as a circular orbit with respect to Edelbaum's analysis.

The variation of Hamiltonian is considered for many cases with respect to performance indexes. Minimum-time, Minimum-effort and minimum-effort-time performance indexes which they are studied for non mass variant, and results are examined for many maneuvers from LEO to GEO.

Edelbum linearizes the Lagrange planetary equations of orbital motion about a circular orbit.

The full set of the Gaussian form of the Lagrangian planetary equations for near-circular orbits is given by:

$$\dot{a} = \frac{2\,a\,f_t}{m\,V} \tag{13}$$

$$\dot{e}_x = \frac{2\,f_t \cos(\alpha)}{m\,V} - \frac{f_n\,\sin(\alpha)}{m\,V} \tag{14}$$

$$\dot{e}_y = \frac{2\,f_t \sin(\alpha)}{m\,V} + \frac{f_n \cos(\alpha)}{m\,V} \tag{15}$$

$$\dot{i} = \frac{f_h \cos(\alpha)}{m\,V} \tag{16}$$

$$\dot{\Omega} = \frac{f_h \sin(\alpha)}{m\,V} \tag{17}$$

$$\dot{\alpha} = n + \frac{2 f_n}{m\,V} - \frac{f_h \sin(\alpha)}{m\,V\,\tan(i)} \tag{18}$$

where a stand for orbit semi major axis, i for inclination and Ω for the right ascension of the ascending node; $e_x = e\,\cos(\omega)$, $e_y = e\,\sin(\omega)$ with e and ω standing for orbital eccentricity and argument of perigee. Finally, $\alpha = \omega + M$ represents the mean angular position, M the mean anomaly, and $n = \left(\frac{\mu}{a^3}\right)^{1/2}$ the orbit mean motion, with μ standing for the Earth gravity constant and for near-circular orbits $V = na = \left(\frac{\mu}{a}\right)^{1/2}$. The components of the thrust vector along the tangent, normal, and out-of-plane directions are depicted by f_t, f_n and f_h with the normal direction oriented towards the center of attraction. If we assume only the tangential and out-of- plane thrust vector Figure 1, and that the orbit remains circular during the transfer, above equations reduces to:

$$\dot{a} = \frac{2\,a\,\xi_t}{V} \tag{19}$$

$$\dot{i} = \frac{\xi_h \cos(\alpha)}{V} \tag{20}$$

$$\dot{\Omega} = \frac{\xi_h \sin(\alpha)}{V} \tag{21}$$

$$\dot{\alpha} = n - \frac{\xi_h \sin(\alpha)}{V\,\tan(i)} \tag{22}$$

If ξ represents the magnitude of the acceleration vector, and β the out-of-plane or thrust yaw angle then $\xi_t = \xi\cos(\beta)$ and $\xi_h = \xi\sin(\beta)$. Furthermore, $\alpha = \omega + M = \omega + \theta^* = \theta$ the angular position when $e = 0$, with $\theta = nt$ and θ^* the true anomaly. If the angle β is held piecewise constant switching sign at the orbital antinodes, then the $\xi_h \sin(\beta)$ terms will have a net zero contribution such that the system of differential equations further reduces to:

$$\dot{a} = \frac{2\,a\,\xi_t}{V} \tag{23}$$

$$\dot{i} = \frac{\xi_h \cos(\alpha)}{V} \tag{24}$$

$$\dot{\theta} = n \tag{25}$$

These equations may now be averaged with respect to true anomaly to obtain the long-period motion using the averaging operator.

$$\langle Z \rangle = \frac{1}{2\pi} \int_0^{2\pi} z \, d\theta \tag{26}$$

Assuming that the out-of-plane thrust angle β is a function of true anomaly only and the change in elements during each orbit is small, and with respect to θ and by holding ξ, β and V constant the averaging operator may be applied to above equations to yield:

$$\int_0^{2\pi} \left(\frac{di}{dt} \right) d\theta = \frac{2\,\xi \sin(\beta)}{V} \int_{-\pi/2}^{-\pi/2} \cos(\theta) \, d\theta \tag{27}$$

$$2\pi \frac{di}{dt} = \frac{4\,\xi \sin(\beta)}{V} \tag{28}$$

$$\frac{di}{dt} = \frac{2\,\xi \sin(\beta)}{\pi\, V} \tag{29}$$

From the energy equation $\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$, and with Eq. (23) used to eliminate the semi major axis:

$$dV = -\left[\frac{\mu}{2\, V a^2} \right] da = -\xi \cos(\beta) \, dt \tag{30}$$

$$\frac{dV}{dt} = -\xi \cos(\beta) \tag{31}$$

Let the system equations be given by Eq. (29) and (31), with variables i and V as the states variables and β as the control variable [6, 15].

$$\begin{cases} \frac{di}{dt} = \frac{2\,\xi \sin(\beta)}{\pi\, V} \\ \frac{dV}{dt} = -\xi \cos(\beta) \end{cases} \tag{32}$$

MIN TIME LOW-THRUST ORBITAL TRANSFER

In this part, transfer problem is cast as minimum time problem between initial and final conditions i_0, V_0 and i_f, V_f respectively. The variational Hamiltonian is then given by:

$$H = 1 + \lambda_i \left(\frac{2\,\xi\, \sin(\beta)}{\pi\, V} \right) + \lambda_v \left(-\,\xi\, \cos(\beta) \right) \tag{33}$$

The performance measure is simply: $J = \int L \, dt$ with $L = 1$. Using the costate equation the Euler-Lagrange differential are given by:

$$\frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial i} = 0 \tag{34}$$

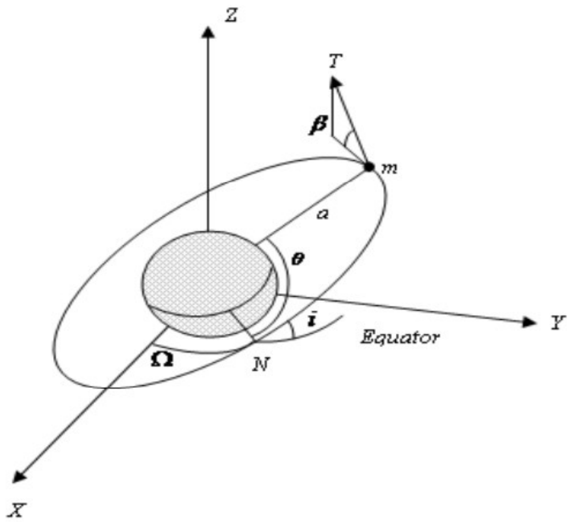

Figure 1. Schematic diagram of transfer.

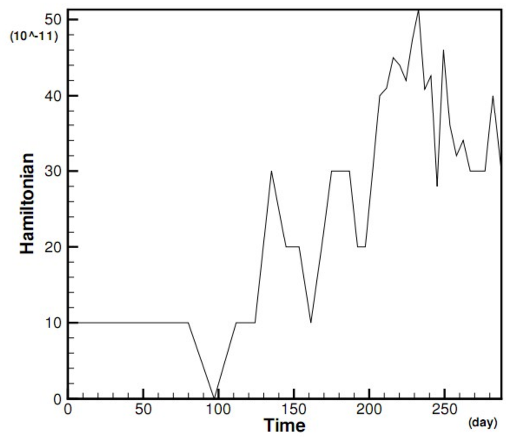

Figure 2. Hamiltonian history for min-time expenditure.

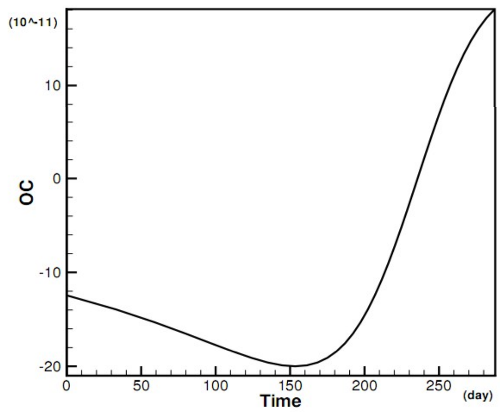

Figure 3. Optimality condition history for min-time expenditure.

$$\frac{d\lambda_v}{dt} = -\frac{\partial H}{\partial V} = \frac{2\,\lambda_i\, \xi\, \sin(\beta)}{\pi V^2} \tag{35}$$

due to Eq. (34), λ_i is a constant. From Eq. (8) the optimal condition is:

$$\frac{\partial H}{\partial \beta} = \frac{2\,\lambda_i\,\xi\,\cos(\beta)}{\pi\,V} + \lambda_v\,\xi\sin(\beta) = 0 \tag{36}$$

Nothing right hand side of system equation is not an explicit function of time, hence H is not explicit function of time, so Hamiltonian is constant on the optimal trajectory and $\dot{H} = 0$. In the other words problem is autonomous and Hamiltonian is constant for an optimal path; since the final time is not specified, the constant must be zero [5, 12]. To illustrate this results and show accuracy of solutions, Figures (2-4) are sketched for boundary conditions of scenario III of Table 1. It is evident that history of Hamiltonian and optimality condition are vanished.

$$H(t) = 0 \tag{37}$$

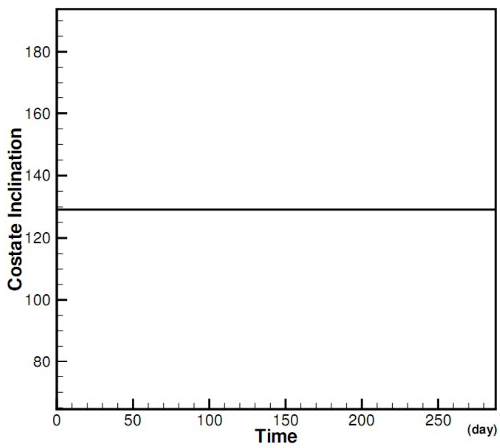

Figure 4. Costate inclination history for min-time expenditure.

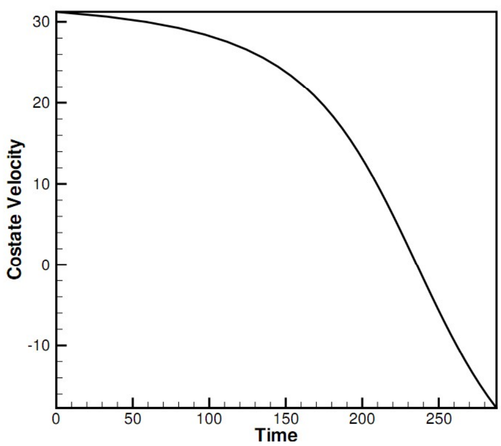

Figure 5. Costate velocity history for min-time expenditure.

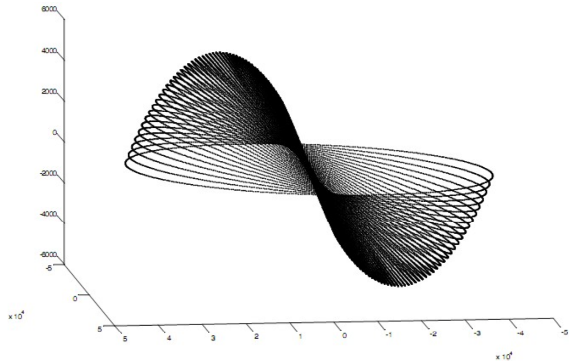

Figure 6. Optimal orbital maneuver from LEO to GEO.

Above conditions and optimality relation are leading to bellows method by considering some proper trigonometric expressions and Eqs. (33, 36).

$$H.(-\cos(\beta)) = \left(1+\lambda_i\left(\frac{2\xi\sin(\beta)}{\pi\,V}\right)+\lambda_v\,(-\xi\cos(\beta))\right)$$
$$.\,(-\cos(\beta))=0 \tag{38}$$

$$\frac{\partial H}{\partial \beta}.\sin(\beta) = \left(\frac{2\,\lambda_i\,\xi\,\cos(\beta)}{\pi\,V} + \lambda_v\,\xi\sin(\beta)\right).\sin(\beta) = 0 \tag{39}$$

By adding Eq. (40) and Eq. (41):

$$H.(-\cos(\beta)) + \frac{\partial H}{\partial \beta}.\sin(\beta) = 0 \tag{40}$$

Hence there is no need to integrate and simply costates are obtained.

$$\begin{cases} \lambda_i = \frac{-1}{2}\,\frac{\pi\ V\sin(\beta)}{\xi} \\ \lambda_v = \frac{\cos(\beta)}{\xi} \end{cases} \tag{41}$$

It should be noted, right hand side of system equation is not an explicit function of time, and the time derivatives appearing in the governing equation can be written with respect to β. In this way, now β becomes the independent variable.

$$\frac{d\beta}{dt} = \frac{\frac{d\lambda_v}{dt}}{\frac{d\lambda_v}{d\beta}} = \frac{\sin(\beta)\,\xi}{V} \tag{42}$$

One can find implicit relation for β and t from Eq. (42), so new system equations are given by:

$$\begin{cases} \frac{di}{d\beta} = \frac{2}{\pi} \\ \frac{dV}{d\beta} = -\frac{V\ \cos(\beta)}{\sin(\beta)} \end{cases} \tag{43}$$

Due to the simple form of Eq. (43), it is integrated to yield the results as a function of the control angle β,

$$V = \frac{\sin(\beta_f)\,V_f}{\sin(\beta)} \quad or \quad V = \frac{\sin(\beta_0)\,V_0}{\sin(\beta)} \tag{44}$$

$$i = i_f + \frac{2(\beta - \beta_f)}{\pi} \tag{45}$$

Obviously for explicit results, it is needed to specify the values of β_0 and β_f. This can be accomplished through using the known initial and terminal conditions and solving a set of non-linear algebraic equations. For example, if conditions are considered as first scenario of Table 1, initial and final control angles are achieved as $\beta_0 = 21.9911°$ and $\beta_f = 66.7838°$ and $\Delta\beta = 44.79270°$. These results are the same and identical with results of Ref. [5] for min-time performance expenditure. But in this work calculations are simplified and reduced to achieve exact closed form solutions for low-thrust orbital transfer. As mentioned, solutions have been expressed in terms of β simply without time interference and it is important note because low-thrust transfers have long transfer time. Figure 6 shows how optimal orbital maneuver accomplished around the Earth. If the problem has no explicit depends on time, then time to go, is really the important time [12]. Next step, time to go is derived with respect to $\dot{\beta}$. Finally Low-thrust total transfer time for first scenario of Table 1 is 191.2738 days. To show the results of this part Figures 7-10 for third scenario of Table 1 are sketched and Figure 11 represents changes of acceleration magnitude vis-a-vis time duration. Figure (11) shows how total time reduces when acceleration increases.

$$Time\,to\,go = \frac{V_f \sin(\beta_f - \beta)}{\xi \sin(\beta)} \tag{46}$$

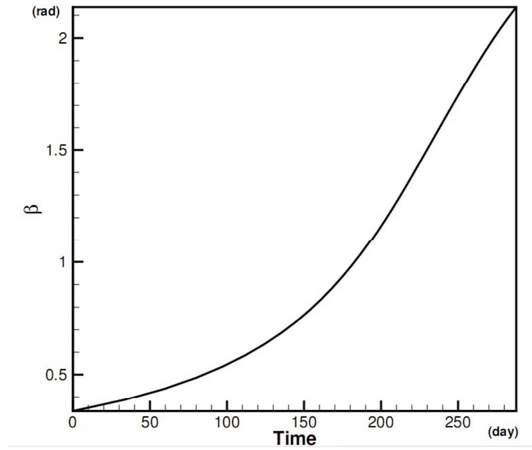

Figure 7. Optimal thrust angle history for min-time expenditure.

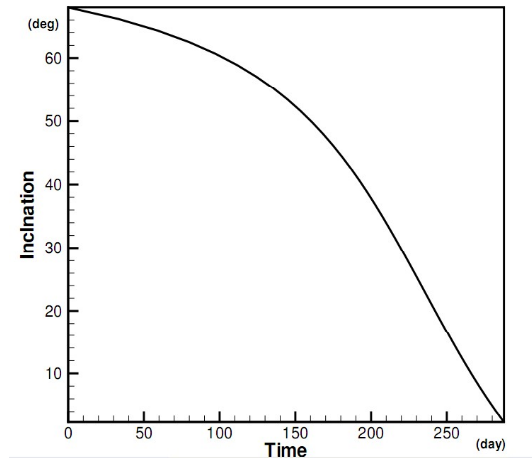

Figure 8. Optimal inclination history for min time expenditure.

CONCLUSION

This paper describes the solutions of an optimal orbital transfer. The dynamics incorporate low-thrust transfer from low earth orbits to high earth orbits by considering orbital plane changes that compatible with the circular maneuver assumptions. The optimal solutions to the min-time, constant-thrust maneuvers have some interesting properties. Using the equation of motion in a different form with respect to thrust angle permits the investigation of essentially complicated calculations which have not previously been reported for this problem. The method used to solve the optimal control problem is referred to use some concepts of optimal control theory especially for autonomous system equations and they make clear why this orbital transfer has to be set up as exact closed form solutions. Min-time performance measure is provided in this

Table 1. Parameters of four scenarios, for minimum time low thrust transfer.

Scenario	Scenario I	Scenario II	Scenario III	Scenario IV
Result	$i_0 = 28.5°$ $i_f = 0°$	$i_0 = 45°$ $i_f = 0°$	$i_0 = 68°$ $i_f = 0°$	$i_0 = 87°$ $i_f = 40°$
Final Time(day)	191.2738	236.2708	295.4406	241.8129
β_0(deg)	21.9911	23.9725	19.2437	23.8294
β_f (deg)	66.7838	94.6942	126.1121	97.6943
$\Delta\beta_{Control}$ (deg)	44.79270	70.7216	106.8683	73.8648
ΔV_{total}	5.7841	7.1448	8.9341	7.3124

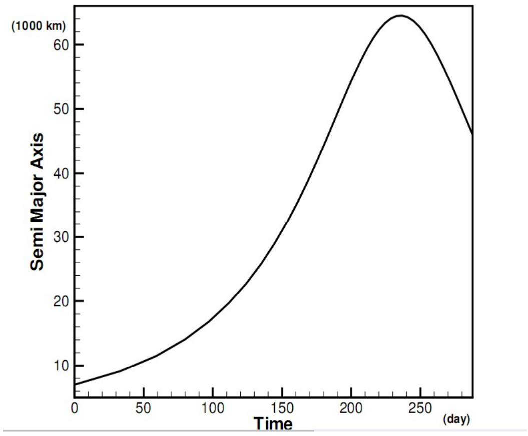

Figure 9. Optimal semi major axis history for min-time expenditure.

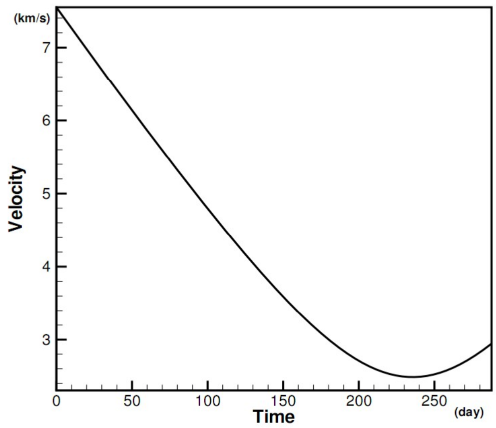

Figure 10. Optimal velocity history for min-time expenditure.

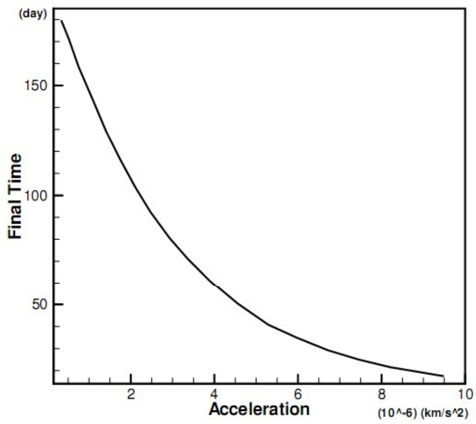

Figure 11. Acceleration magnitude vis-a-vis time duration.

work. The results presented here provide a through foundation for future studies of continuous, low-thrust problems of this type in closed form solutions especially for other performance indexes like min-effort, min-fuel and *etc.*

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