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Natural Periodic Orbit Attitude Behavior Of Satellites In Three-Body Problem In The Presence Of The Oblate Primaries

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ABSTRACT

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The main purpose of this article is to examine the periodic coupled orbit-attitude of a satellite at restricted three body problem considering both primaries oblateness perturbations. The proposed model was based on a simplified coupled model meaning that the time evolution of the orbital state variables was not a function of the attitude state variables. Since, the problem has no closed-formed solution, and the numerical methods must be used, so the problem can have different periodic or non-periodic responses to the initial conditions. The initial guess vector of the coupled model's states was introduced to achieve the optimal initial conditions leading to the periodic responses, and then the P-CR3BP coupled orbit-attitude correction algorithm was proposed to correct this initial guess. Since, the number of periodic solutions is restricted; the suitable initial guess vector as the inputs of the coupled orbit-attitude correction algorithm increases the chances of achieving more accurate initial conditions. The initial guess of orbital states close to the initial conditions of the P-CR3BP periodic orbit, along with initial guess vector of attitude dynamics states with Poincaré mapping was suggested as the suitable initial guess vector of the coupled model.

Introduction

Today, space agencies are increasingly using multi-body dynamic structures for their missions. These missions are carried out with objectives such as astronomical observations [1-3], preparation of human habitats in space, improving the accuracy of pointing on space telescopes and connection [4] to the space station. When the spacecraft is placed in the multi-body orbital regime and is coupled with it, it may show complex behavior. The problem of three circular limited bodies can be a suitable approximation of the mentioned multi-body structures in which the spacecraft floats in the gravitational field of two main planets and its presence does not affect the movement of the main attractors. Considering the mass of the spacecraft is insignificant compared to the mass of the main absorbers, the reason for the lack of influence of the motion of these absorbers from the spacecraft. Therefore, expressing the fundamental behaviors of the orbit-state coupling problem in this research requires a correct understanding of the state dynamics of the space vehicle when it is coupled with the orbital regime of the problem of three circular bounded bodies. Considering the mentioned orbital regime, the initial research by Kane and Marsh was done on the positional dynamics of satellites with different bodies [4-6]. In this research, it was assumed that the satellites were artificially placed in the equilibrium points of the three-body system. Further, studies on this field led to the introduction of Euler's parameters, quaternions and Poincaré mapping in the expression of satellite state dynamics. In these studies, it was assumed that the

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satellite is placed on the Lagrangian points of the three-body problem [7]. Wong, Patil and Misra investigated the effect of the gravitational torque on the spacecraft in orbits around the equilibrium points of the Earth-Sun system [8]. They used the Lyapanov circuits as reference trajectories expressed linearly for their research. Another simplification of this problem was made by using Hill's equations for dumbbell-shaped satellites that were placed in the Lyapanov orbits of the Earth-Moon system [9]. The application of Hill's problem in this research was limited to the condition of proximity of the spacecraft to the smaller main attractor. Gazzetti investigated the state-orbit Kopplink problem numerically by considering Lyapanov orbits as the reference path. In his research, the reference paths were expressed in a non-linear way, but the rotation of the satellite was limited to the orbital plane [10-12]. The use of Poincaré mapping in identifying the initial conditions of the situational parameters in order to meet the alternate answers of the satellite's situation, in addition to the appropriate accuracy, has the advantage of reducing the dimensions of the system to the dimensions of interest in the study and will also avoid complex mathematical calculations. Periodic and quasi-periodic structures in this sample of problems can be useful in maintaining the station or transmitting data continuously. In consideration of alternating responses, they can contribute to the understanding of the satellite's positional dynamics when coupled with the three-body orbital regime. In the general case, the matching of the starting point and the end point of the solution in terms of time are known as alternating responses. Considering the mentioned conditions will lead to a kind of simplification, according to which, the double boundary condition problem will be transformed into a single boundary condition problem. Adding spatial disturbances will be efficient in modeling the environment of the problem as accurately as possible. Singh performed orbital analysis in the regime of three circular bodies by considering the cooked Earth in the Earth-Moon system [13]. Sirvastava and Kumar performed the orbits of the system of three circumscribed bodies in the presence of the earth's curvature and the effects of the sun's radiation pressure in the earth-sun system [14]. They used Lagrangian mechanics in their research to obtain the equations of orbital motion. Marklos and Papadakis studied the nonlinear stability of the satellite around the Lagrangian points of the Earth-Moon system by considering the perturbation of the Earth's curvature [15]. This research was completed by Singh by considering spacecraft with variable mass [16]. Orbital analyzes in the problem of three limited circular bodies by considering the disturbances of the bigger planet's maturation, solar radiation pressure and considering the effect of the fourth body are worth mentioning in this context [17-19]. It should be noted that in all these researches, the effect of spatial disturbances has been investigated only on the problem of orbital motion. The main goal of this article is to find the alternating mode responses of the satellite in the alternating Lyapanov orbits of the problem of three circular bounded bodies in the presence of cooking disturbances. Since the studied problem does not have any closed-loop solution, therefore it is necessary to use numerical methods, so the problem can have non-alternating solutions or Be intermittent. In this regard, Poincaré mapping will be used to identify the appropriate initial conditions of the situational parameters. The islands formed in the maps created by the Poincaré mapping are identified as appropriate initial guesses of the state dynamic state parameters. Also, the initial conditions of the orbital state parameters in the turbulent environment will be chosen near the initial conditions of the Lyapanov orbits identified in the research of Abbasei et al. [20,21]. Investigating the impact of disturbances will lead to a better understanding of the natural movement of the satellite, which will have an impact on the success of a mission. First, in section 2, we will introduce the frameworks needed to describe the problem. Then, in section 3, the equations of the satellite's orbital motion in the circular bounded body problem in the presence of the perturbations of both main planets will be derived using Lagrangian mechanics. Also, the equations of state motion will be described using Newton's second law in the form of angular velocity in section 4.

Requirements frames

When the two main attractors move on a certain circular orbit, it is possible to define the coordinates that are constant with respect to their movement. The center of the said coordinates is located at the center of mass of the main absorbers and this coordinate rotates with a constant angular velocity Ω, which is equal to the average movement of the main absorbers. These coordinates, which are known as rotating coordinates, are defined by the vectors $\mathbf{r}(\hat{x},\hat{y},\hat{z})$ in such a way that its $\hat{Z}$ component is perpendicular to the plane of movement of the main attractors and objects. m1 and m2 always remain on the vector x̂ these coordinates. Also, the inertial coordinates $\boldsymbol{I}(\hat{X},\hat{Y},\hat{Z})$are defined in such a way that at the initial time t=0 it is parallel to the rotating coordinates and its vector $\hat{Z}$ is always in the direction of the vector ẑcoordinates The rotor should be parallel and perpendicular to the plane of movement of the main attractors. Also, a physical coordinate connected to the body of the satellite is needed, which is represented by the vectors $\boldsymbol{b}(\hat{b}_1,\hat{b}_2,\hat{b}_3)$Figure 1 shows the defined coordinates.

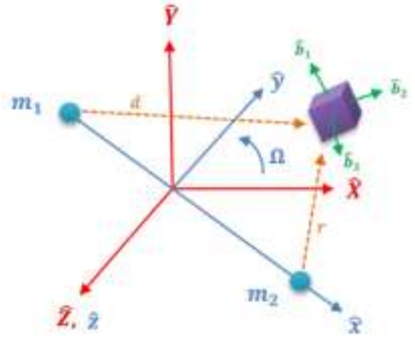


Fig1. CRTB requirement frames

Orbital equation of motion

Consider two flat planets with masses m1 and m2 that move in a circular orbit only under the influence of mutual gravity (m1>m2). We have added a satellite with a very small mass compared to the mass of the mature planets to the mentioned system and we are interested in describing its motion equations in the description system. Abbas Ali et al. derived the equations of satellite orbital motion in the problem of three bounded circular bodies in the presence of perturbations of the main planets [20, 21]. In their research, they considered the regional harmonic effect of the planets. For this reason, in this article, further explanations about the process of deriving these equations are avoided and it is directly mentioned to the equations given in the mentioned reference that these equations will be written in the dimensionless form as follows:

$$
\begin{aligned}
\ddot{x} &= n^2x + 2nv_y + \mu(\mu + x - 1)R^* - (\mu-1)(x+\mu)D^* \\
\ddot{y} &= n^2y - 2nv_x + \mu yR^* - (\mu-1)yD^* \qquad (1) \\
\ddot{z} &= \mu zR^* - (\mu-1)zD^*
\end{aligned}
$$

In the group of equations, R* and D* are equal to:

$$
R^* = \left[\frac{1}{r^3} + \frac{3A_2^{(2)}}{2r^5}\right] \qquad (2)
$$

$$
D^* = \left[\frac{1}{d^3} + \frac{3A_2^{(1)}}{2d^5}\right]
$$

$$
\begin{aligned}
d &= \sqrt{(x+\mu)^2 + y^2 + z^2} \qquad (3) \\
r &= \sqrt{(x-1+\mu)^2 + y^2 + z^2}
\end{aligned}
$$

Also $\mu = \frac{m_2}{m_1+m_2}$ representative of the mass constant and the average amount of movement will be obtained from the following relationship [22].

$$
n = \sqrt{1 + 3(J_2^{(1)} + J_2^{(2)})} \qquad (4)
$$

The second regional harmonics of the planets are denoted by symbols $J_2^{(1)}$ and $J_2^{(2)}$. The flattening coefficients of these planets are indicated by the symbols $A_2^{(1)}$ and $A_2^{(2)}$, which for each planet is equal to the product of the second regional harmonic multiplied by the square of the equatorial radius.

Attitude dynamics:

The fundamental equations of rotational motion of a rigid body can be derived using Newton's second law in the form of angular velocity. The vector of external torques applied to the body of the rigid satellite, expressed in the body frame $\hat{b}$ which is connected to the body of the rigid body relative to the inertial frame, can be written as

$$
\boldsymbol{M}^B = M_1\hat{b}_1 + M_2\hat{b}_2 + M_3\hat{b}_3 \qquad (5)
$$

In this case, the law of angular velocity, which in fact represents the equations predicting the rotational dynamics of a rigid body around its center, will be written as follows [23,24]:

$$\begin{aligned} I_1\dot{\omega}_1 &= -(I_3 - I_2)\omega_2\omega_3 + M_1 \\ I_2\dot{\omega}_2 &= -(I_1 - I_3)\omega_1\omega_3 + M_2 \\ I_3\dot{\omega}_3 &= -(I_1 - I_1)\omega_1\omega_2 + M_3 \end{aligned} \quad (6)$$

In the system of equations (6), the symbol I_i represents the main moments of inertia corresponding to the body axes $b\hat{}\_i$. Also, the symbol M_i represents the vector of external torques applied to the body of the satellite object. In this case, the equations of time rate change of satellite angular velocities in this problem will be summarized as follows [25]:

$$\dot{\omega}_1 = \frac{I_3 - I_2}{I_1}\Big(\frac{3(1-\mu)}{d^3} g_2 g_3 + \frac{3\mu}{r^3} h_2 h_3 - w_2 w_3\Big) \quad (7)$$

$$\dot{\omega}_2 = \frac{I_1 - I_3}{I_2}\Big(\frac{3(1-\mu)}{d^3} g_1 g_3 + \frac{3\mu}{r^3} h_1 h_3 - w_1 w_3\Big)$$

$$\dot{\omega}_3 = \frac{I_2 - I_1}{I_3}\Big(\frac{3(1-\mu)}{d^3} g_1 g_2 + \frac{3\mu}{r^3} h_1 h_2 - w_1 w_2\Big)$$

where h_i represents the image vector that connects the spacecraft and the larger main attractor m1 and g_i represents the image vector that connects the spacecraft and the smaller main attractor m2 expressed in the body frame. According to the introduction of the cosine direction matrix of the conductor, the vector of images h_i and g_i is defined as a function of the instantaneous position and orientation of the satellite:

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \boldsymbol{A}_{\hat{b}.\hat{\imath}}\boldsymbol{A}_{\hat{\imath}.\hat{r}}\frac{\boldsymbol{d}}{d} = \boldsymbol{A}_{\hat{b}.\hat{\imath}}\boldsymbol{A}_{\hat{\imath}.\hat{r}}\frac{1}{d}\begin{bmatrix} x+\mu \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \boldsymbol{A}_{\hat{b}.\hat{\imath}}\boldsymbol{A}_{\hat{\imath}.\hat{r}}\frac{\boldsymbol{r}}{r} = \boldsymbol{A}_{\hat{b}.\hat{\imath}}\boldsymbol{A}_{\hat{\imath}.\hat{r}}\frac{\boldsymbol{1}}{d}\begin{bmatrix} x-1+\mu \\ y \\ z \end{bmatrix} \quad (8)$$

In equations 8, the body-inertial and rotational-inertial transfer matrices are represented by the symbols $A_{b\,\hat{}\,.i\,\hat{}}$ and $A_{i\,\hat{}\,.r\,\hat{}}$, respectively.

$$\boldsymbol{A}_{\hat{\imath}.\hat{r}} = \begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$\boldsymbol{A}_{\hat{b}.\hat{\imath}} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 + q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \quad (10)$$

where $q = [q_1 q_2 q_3 q_4]$ represents the quaternion vector. In order to propagate quaternions in time, equations (10) are used.

$$\begin{aligned} \dot{q}_1 &= \frac{1}{2}(\omega_3 q_2 - \omega_2 q_3 + \omega_1 q_4) \\ \dot{q}_2 &= \frac{1}{2}(-\omega_3 q_1 + \omega_1 q_3 + \omega_2 q_4) \\ \dot{q}_3 &= \frac{1}{2}(\omega_2 q_1 - \omega_1 q_2 + \omega_3 q_4) \\ \dot{q}_3 &= \frac{1}{2}(-\omega_1 q_1 - \omega_2 q_2 - \omega_3 q_3) \end{aligned} \quad (11)$$

The system of equations (1), (7) and (11) are known as the governing equations for the description of the simple coupling of the orbit-state of the satellite in the problem of three bounded bodies in the presence of the disturbances of the cooking of both main absorbers. Due to the lack of any loop-closed solution for the extracted equations, it is necessary to use numerical solution methods. Therefore, solving the problem will have a strong dependence on the initial conditions of the circuit-state parameters. According to the main goal of this article to identify simultaneous circuit-state alternating responses in the described problem and also the limitation in the number of these responses, the problem will be the need for very precise initial conditions in order to meet the alternating responses. For this purpose, an algorithm called circuit-state modification algorithm will be developed, which will be responsible for correcting the initial guesses of the circuit-state parameters.

Orbit-Attitude Correction:

The main goal of this article is to identify and analyze the intermittent responses of the satellite orbit-state coupling in the problem of three circular bounded bodies in the presence of the perturbations of both main planets. In order to meet these answers, the number of which is very limited, the problem requires accurate and appropriate initial conditions of state parameters. For this purpose, in this article, an algorithm called circuit-state modification algorithm will be developed. The main task of this algorithm is to modify the initial guesses of the state-orbit state parameters to accurate initial conditions, the use of which leads to meeting the simultaneous alternating responses of the state-orbit state. In this article, the modified condition vector and the initial guess vector of these condition parameters will be shown with the symbols ς_0^* and ς_0, respectively. Now it is assumed that the initial guess vector of state parameters is as follows:

$$\boldsymbol{\varsigma_0} = [x_0(t_0)\,, y_0(t_0)\,, \dots, v_{0_z}(t_0)\,, q_{1_0}(t_0)\,, \dots, q_{4_0}(t_0)\,, \omega_{1_0}(t_0)\,, \dots, \omega_{3_0}(t_0),] \quad (12)$$

This vector consists of two sections of orbital and position parameters, where t_f is the suggested time of the periodic period. In this case, the meeting of the circuit-state alternating responses will be required to establish the constraint vector τ:

$$\boldsymbol{\tau} = \begin{bmatrix} x(t_0) - x(t_f) \\ y(t_0) - y(t_f) \\ z(t_0) - z(t_f) \\ v_x(t_0) - v_x(t_f) \\ v_y(t_0) - v_y(t_f) \\ v_z(t_0) - v_z(t_f) \\ \omega_1(t_0) - \omega_1(t_f) \\ \omega_2(t_0) - \omega_2(t_f) \\ \omega_3(t_0) - \omega_3(t_f) \\ q_1(t_0) - q_1(t_f) \\ q_2(t_0) - q_2(t_f) \\ q_3(t_0) - q_3(t_f) \\ q_4(t_0) - q_4(t_f) \end{bmatrix} = 0 \quad (13)$$

where t_0 is equal to the initial time and t_f is equal to the periodic time, which is equivalent to the periodic time of Lyapanofi's alternating orbits of the problem of three bounded bodies in the presence of the aforementioned disturbances. The information of the mentioned circuits is available in the article of Abbasali et al. [20-21]. In the following, the multivariate Newton-Raphson method is used in the following form in order to identify the appropriate initial conditions that apply to clause (13):

$$\varsigma^{i+1} = \varsigma^i - [J_{13\times13}(\varsigma)^i]^{-1}[F_{13\times1}(\varsigma)^i] \quad (14)$$

Initial guess of the orbital states

In this article, it is suggested to choose suitable initial guesses of orbital parameters near the initial conditions of Lyapanov orbits of the mentioned problem [20]. The idea of this proposal originates from the fact that the knowledge of the periodic circuit in which the state behavior is supposed to be periodic is useful in speeding up the convergence process of the circuit-state modification algorithm. In other words, in the process of solving the problem, the alternating circuit in which the behavior of the situation is supposed to be alternating should be identified first. In this way, in the first step, the said circuit is identified through a circuit differential correction algorithm [20], [26]. Considering that the circuit-state modification algorithm simultaneously corrects the initial conditions of the circuit and state parameters, therefore it is not possible to keep the initial conditions of the periodic circuit identified in this algorithm constant. But it is possible to guide the algorithm in the simultaneous convergence to the selected circuit and the alternating behavior of the state in this circuit by choosing the initial circuit guesses, close to the initial conditions of the identified circuit. It should also be noted that the implementation of the Poincaré mapping in order to identify the initial guesses of the state parameters in Section 5-1 requires that the alternating circuit in which the state response is supposed to be alternating is also identified. Therefore, according to the knowledge of the mentioned circuit, choosing the initial guesses of the circuit state parameters close to the initial conditions of the identified alternating circuit will be useful in the accuracy and convergence speed of the circuit-state modification algorithm. Lyapanov orbits are two-dimensional and alternating orbits that lie in the plane of motion of the main planets. So far, many researches have been done on identifying these circuits. Abbasali et al. [20, 21], in their research, developed an algorithm called the orbital correction algorithm in order to identify the initial conditions of Lyapanovi alternating orbits in the three-body problem without perturbations and also in the three-body problem in the presence of perturbations. The initial conditions of Lyapanov circuits are given in the mentioned studies. Appropriate initial guesses of circuit-state modification algorithm are suggested by assuming a small error on these initial conditions.

Initial guess of the attitude parameters:

In this article, it is suggested to use Poincaré mapping in order to extract initial guesses of state parameters. Poincaré mapping is a suitable tool to record the dynamic structures of an n-dimensional system, such as periodic or quasi-periodic structures, in the form $\dot{x} = f(x)$, whose main basis is the use of the dynamic flow of the desired system. To implement this method, we first define an n-1 dimensional cross section perpendicular to the dynamic flow and spread our initial guesses on it. Then, using the governing equations in each stage, we will release the initial guesses and record and display the dynamic flow encounter with the defined cross-sectional area (Figure 2).

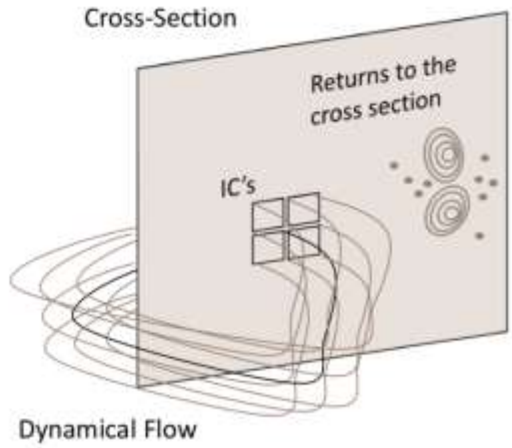


Figure 2. Schematic representation of the Poincaré map [27]

Note that the defined cross section can be any combination of system state parameters. Specifically, periodic and quasi-periodic structure intervals of a two-dimensional structure appear along a closed curve on the map created on the cross section. These closed curves do not have a separate structure from each other but meet in islands and the centers of the formed islands will be considered as periodic responses. In order to better understand the implementation of this method, it will be given in the form of an example. Consider a symmetric disk-shaped satellite with the following moment of inertia characteristics:

$$K_1 = \frac{I_3 - I_2}{I_1} \qquad K_2 = \frac{I_1 - I_3}{I_2} = -K_1 \quad K_3 = \frac{I_2 - I_1}{I_3} = 0 \quad (I_2 = I_1) \tag{15}$$

where K_1, K_2, K_3 are the ratios of the main moments of inertia corresponding to the directions of the body frame bˆ. Since the condition $K_2 = -K_1$ is established in a symmetric disk-shaped satellite, the moment of inertia ratio parameter is defined as $K = K_2 = -K_1$ in this article. In the following, an alternating orbit obtained in past researches will be selected as the reference path of the satellite movement. Now, we consider an interval like [4, 4] for initial guesses of satellite angular velocities. Now, in order to start the process, we choose a number from the mentioned interval and change it in each step with a desired step. The updating of other state parameters will be done in each step using dynamic diffusion equations. In each step, the desired parameters of the dynamic flow will be recorded and displayed on the selected cross section. For example, in Figure 3, the quaternion mode parameters q_2 and angular velocity ω_2 can represent a type of satellite's rotational motion.

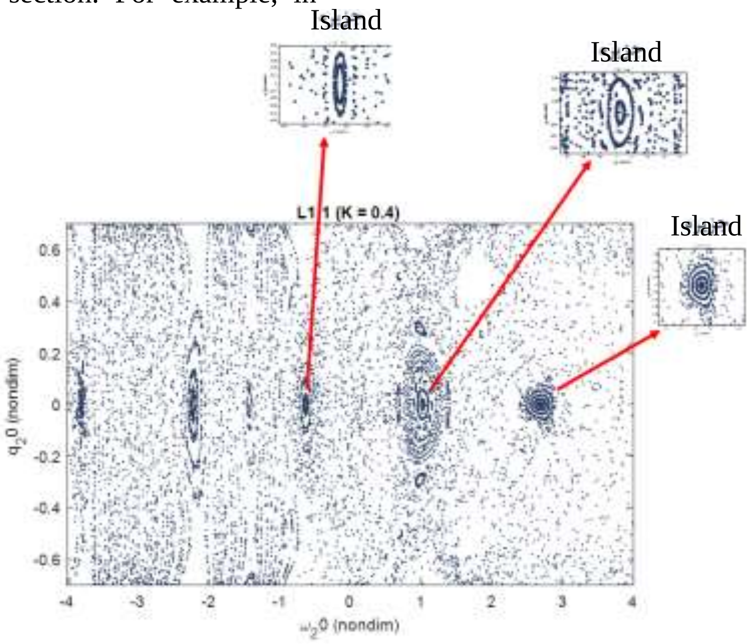


Figure 3. An example of using Poincaré mapping to identify initial guesses of quaternion state parameters q_2 and angular velocity ω_2 for a disk-shaped satellite with inertia ratio K=0.4 in the Earth-Moon system.

Results:

In this article, the planets Earth and Moon are used as the main planets of the problem of three limited bodies, whose constants are given in Table 1:

Table 1. Constants of the mature Earth-Moon system

$\mu_{Earth-Moon}$	$\mathbf{0.012}$
$J_2^{(earth)}$	1.0826×10^{-3}
$J_2^{(moon)}$	2.0323×10^{-4}
$D_{Earth-Moon}$ (km)	384400
$R_{e_{Earth}}$ (km)	6378.1
$R_{e_{Moon}}$ (km)	1738.1
$R_{p_{Earth}}$ (km)	6356.8
$R_{p_{Moon}}$ (km)	1736.1

As mentioned, the circuit-state modification algorithm requires accurate and appropriate initial guesses to converge to the desired initial conditions. The use of these initial conditions will lead to the meeting of simultaneous circuit-state alternating responses in the mentioned problem. In part 1-5, it was suggested that the appropriate initial guesses of orbital parameters should be chosen near the initial conditions of the Lyapanov orbits of the mentioned problem. The initial conditions of the Lyapanov orbits of the baked Earth-Moon problem are available in past researches [20, 21]. Therefore, the appropriate initial guesses of the orbital state parameters are selected with a small error of 0.3 (dimensionless) compared to these initial conditions. Next, among the existing circuits, some of which are drawn in Figure 4, one circuit is selected as the reference path. The initial guesses of the circuit parameters are selected by taking into account the mentioned error on the initial conditions of this circuit.

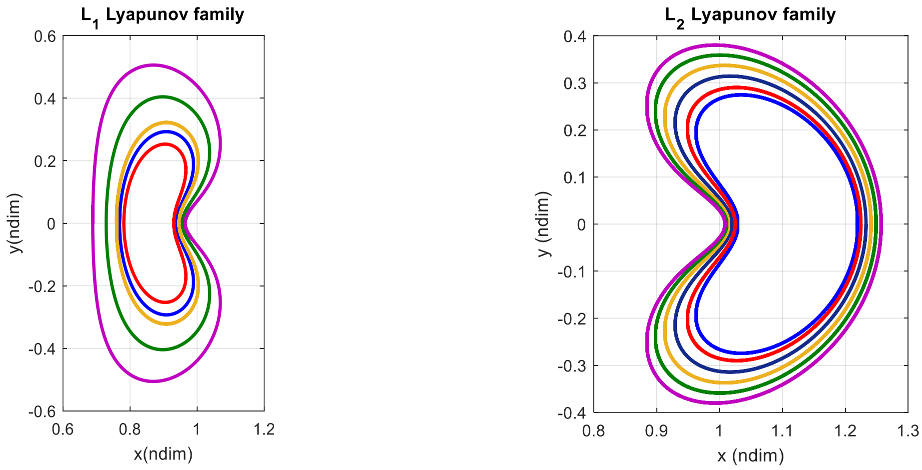


Figure 4. An example of the Lyapanov orbits of the Earth-Moon system around the equilibrium points L_1 and L_2

In the following, in order to identify the initial conditions of the state dynamic state parameters, the Poincaré mapping approach is used. The initial condition vector of the parameters of the satellite's mode dynamic state includes 7 elements in the form $[q_{1_0} q_{2_0} q_{3_0} q_{4_0} \omega_{1_0} \omega_{2_0} \omega_{3_0}]$. These 7 elements should be selected by Poincaré mapping. The results obtained in this research showed that in order to find the appropriate initial conditions for the state dynamics parameters in the alternating Lyapanov orbits to meet the periodic responses of the satellite state in the form [1 0 0 0 0 0 ω_(2_0) 0] It will be that the value of ω_{0_2} should be identified from the Poincaré map. For this reason, in this section, only Poincaré maps are used to identify ω_{0_2}. Finally, the initial guesses of the circuit-state parameters are entered into the circuit-state modification algorithm as input. In the following, as an example, the first orbit of the Lyapanov family around the equilibrium point L_1 is chosen as the reference path. The Poincaré map for a disk-shaped satellite with an inertia ratio of K=0.05 with the assumption of a selected orbit is drawn in Figure 5:

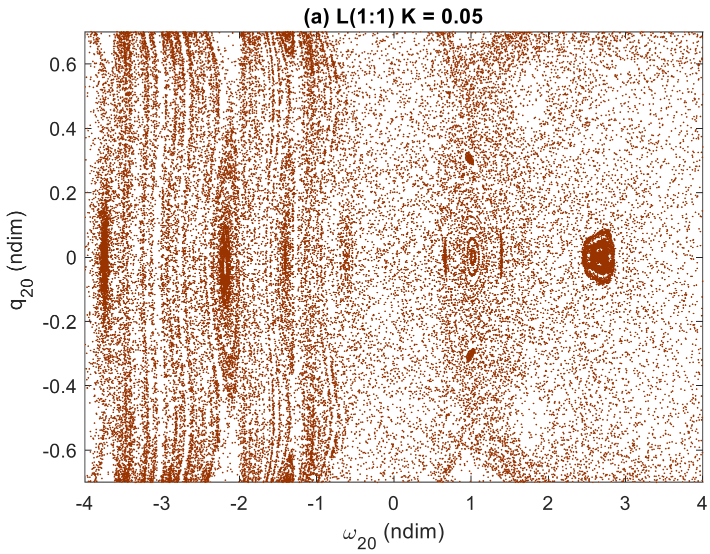


Figure 5. Poincaré mapping in order to identify initial guesses of the mode parameters for a disk-shaped satellite with inertia ratio K=0.05 in the Earth-Moon system.

In the following, Table 2 contains examples of selected initial guesses according to the Poincaré map drawn in Figure 5, and the initial conditions modified by the circuit-state modification algorithm. In this table, the $a-th$ orbit from the family of Lyapanov orbits around the equilibrium point b is displayed with the symbol L(b:a)

Table 2. Examples of selecting the initial guesses of the state-orbit parameters according to the Poincaré map drawn in Figure 2 for a disc-shaped satellite with inertia ratio K=0.05 in the first orbit of the Lyapanov family around the equilibrium point L_1 in the Earth-Moon system.

Orbit	x_0 $(ndim)$	v_{y_0} $(ndim)$	q_{1_0} $(ndim)$	q_{2_0} $(ndim)$	q_{3_0} $(ndim)$	q_{4_0} $(ndim)$	ω_{1_0} $(ndim)$	ω_{2_0} $(ndim)$	ω_{3_0} $(ndim)$	T $(ndim)$
$\varsigma_0(L(1{:}1))$	0.7689000	0.4432	1	0.00000	0.00000	1	0.0000	1.0000	0	3.95
$\varsigma_0^*(L(1{:}1))$	0.7815732	0.4432	0.7071	0.00002	−0.00002	0.7071	−0.00008	1.0101	0	
$\varsigma_0(L(1{:}1))$	0.7689000	0.4432	1	0.000	0.00000	1	0.000	−2.500	0.00000	3.95
$\varsigma_0^*(L(1{:}1))$	0.7815732	0.4432	0.7071	−0.0046	0.00048	0.7071	−0.0029	−2.205	−0.000004	

In order to show the effect of Earth-Moon planetary curvature perturbations in the problem of three bounded bodies, the Poincaré map assuming the same assumptions of the state-orbit state parameters (as the inputs of the Poincaré map) for a disk-shaped satellite with inertia ratio K = 0.04 once considering disturbances (Perturbed) and another time without considering disturbances (Unperturbed) is drawn in Figure 6.

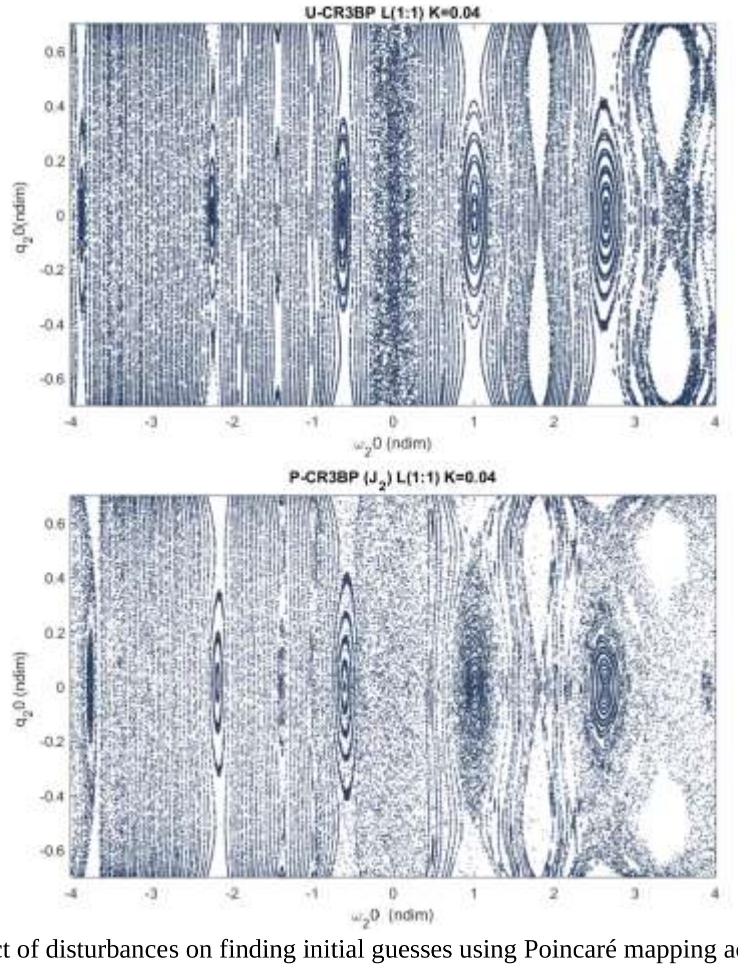


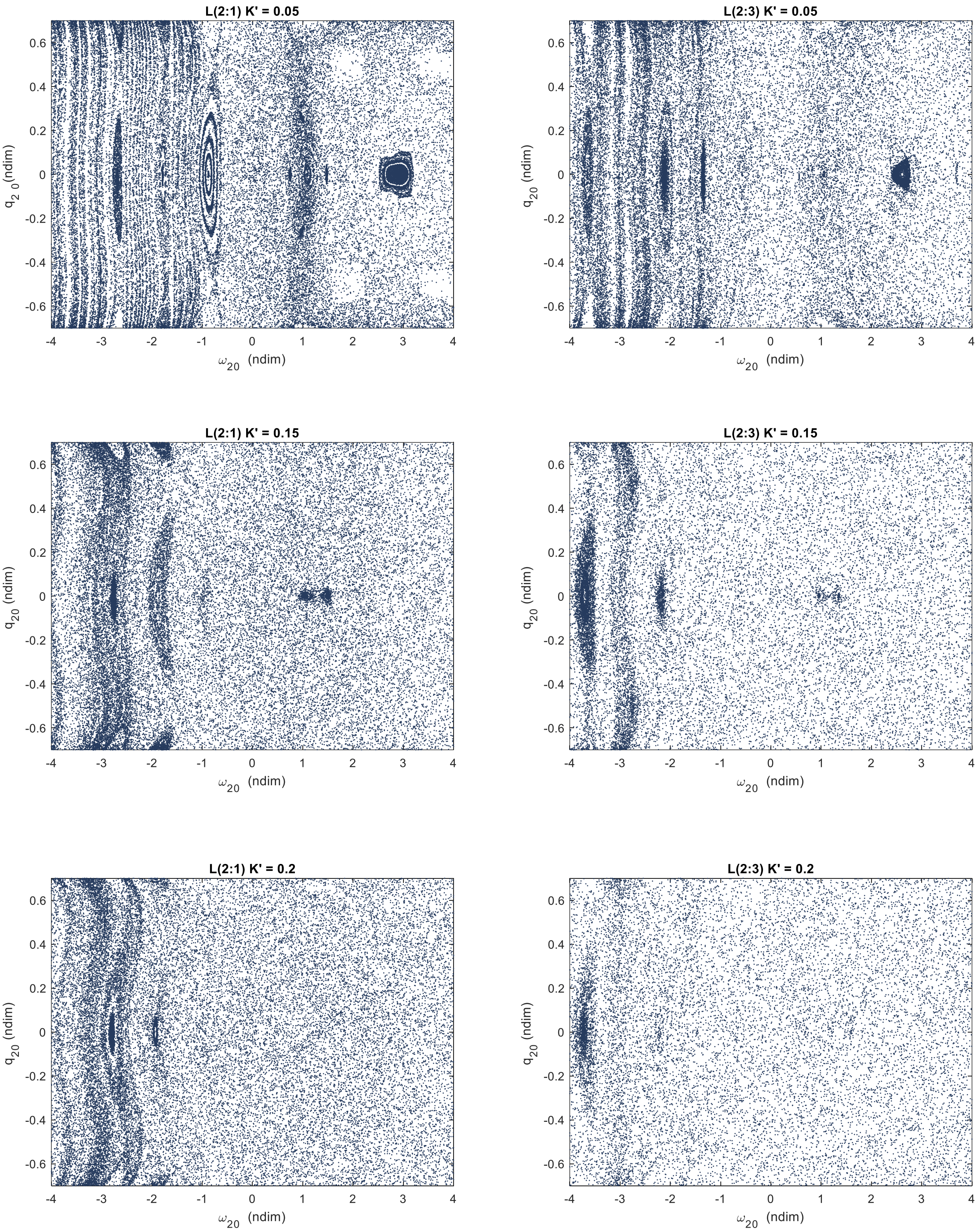
Figure 6. The effect of disturbances on finding initial guesses using Poincaré mapping according to the initial guesses of the parameters of the dynamic flow state by considering a disk-shaped satellite with inertia ratio K=0.04 in the first orbit of the Lyapanov family around the equilibrium point L_1

The change in the pattern of the islands formed in the Poincaré maps drawn in Figure 6, with regard to the same inputs for drawing the map, is a good example of the role and impact of disturbances in the studied problem. As it is clear from this figure, in both models, islands are formed in almost common points, but their shape and pattern are different. In the following, it will be shown that the selection of the same initial guesses in both disturbance and non-disturbance models will lead to different initial conditions in order to match the circuit-state alternating responses in the two mentioned models. In Table 3, some of the same initial guesses extracted from the Poincaré maps drawn in Figure 6 along with the initial conditions modified by the orbit-state modification algorithm are given in two states without disturbance and with disturbance of the Earth-Moon planets. This table also includes the initial guesses and initial conditions of two disturbance and non-disturbance models for other equilibrium points. The symbol P in this table corresponds to the disturbed model and the symbol U corresponds to the unperturbed model.

Table 3. The modified initial conditions of the state-orbit parameters according to the same initial guesses of these parameters for the simple Earth-Moon problem and disturbance by considering a disk-shaped satellite with an inertia ratio of K=0.04 around the linear equilibrium points.

	x_0 (ndim)	v_{y_0} (ndim)	q_{1_0} (ndim)	q_{2_0} (ndim)	q_{3_0} (ndim)	q_{4_0} (ndim)	ω_{1_0} (ndim)	ω_{2_0} (ndim)	ω_{3_0} (ndim)	T (ndim)
ς_0	0.7689	0.4432	1	0	0	1	0	1	0	3.95
$\varsigma_0^{*P}(L^P(1:1))$	0.7815732	0.4432	0.7071	0.00002	−0.00002	0.7071	−0.00008	1.0101	0	
$\varsigma_0^{*U}(L^U(1:1))$	0.7815729	0.4432	0.7083	0.0088	−0.018	0.7055	−0.03	1.02	−0.001	
ς_0	1.22	−0.4275	1	0	0	1	0	1.2	0	4.31
$\varsigma_0^{*P}(L^P(2:1))$	1.219979	−0.4275	0.7	0.0005	−0.0003	0.7141	0	1	0.001	
$\varsigma_0^{*U}(L^U(2:1))$	1.219978	−0.4275	0.721	0.0007	−0.0005	0.6929	0.0002	1.01	0	
ς_0	−1.6068	1.1159	1	0	0	1	0	1.4	0	6.225
$\varsigma_0^{*P}(L^P(3:1))$	−1.06068057	1.1159	0.7071	−0.00004	−0.00004	0.7071	0	1.03	0	
$\varsigma_0^{*U}(L^U(3:1))$	−1.606804	1.1159	0.7072	−0.0002	0.0001	0.7069	0.0008	1.05	−0.002	

As it is clear from the data in Table 3, taking into account the cooking disturbances, in addition to the effect on the pattern of the islands formed in the Poincaré maps (assuming the same inputs of the two models), causes a change in the modified initial conditions of the circuit-state (according to The same initial guesses of the state parameters of the circuit-state) are made in two perturbation and non-perturbation models in order to meet the simultaneous alternating responses of the circuit-state. One of the influential parameters in the convergence of the algorithm to the desired response is the inertia ratio parameter K. The research carried out in this article showed that it is possible that a vector of initial guesses of circuit-state parameters with different values of inertia coefficient leads to the identification of different initial condition vectors of circuit-state parameters in order to meet simultaneous alternating responses of circuit-state or that the problem does not have any alternate solution for certain values of the inertia coefficient. In order to identify the inertia ratios in this article, Poincaré maps were drawn for the Lyapanov circuit family around the equilibrium point of point L_2 for the inertia moment ratios K, from there the results are similar between the circuits in each family. Figure 7 is an example. These maps are given for a Lyapanov orbit L_2 . The left column is drawn for an orbit with a period of 3.7 (dimensionless) and the right column is drawn for an orbit with a period of 4 (dimensionless).


L(2:1) K' = 0.05
L(2:3) K' = 0.05
L(2:1) K' = 0.15
L(2:3) K' = 0.15
L(2:1) K' = 0.2
L(2:3) K' = 0.2
q_{20} (ndim)
ω_{20} (ndim)

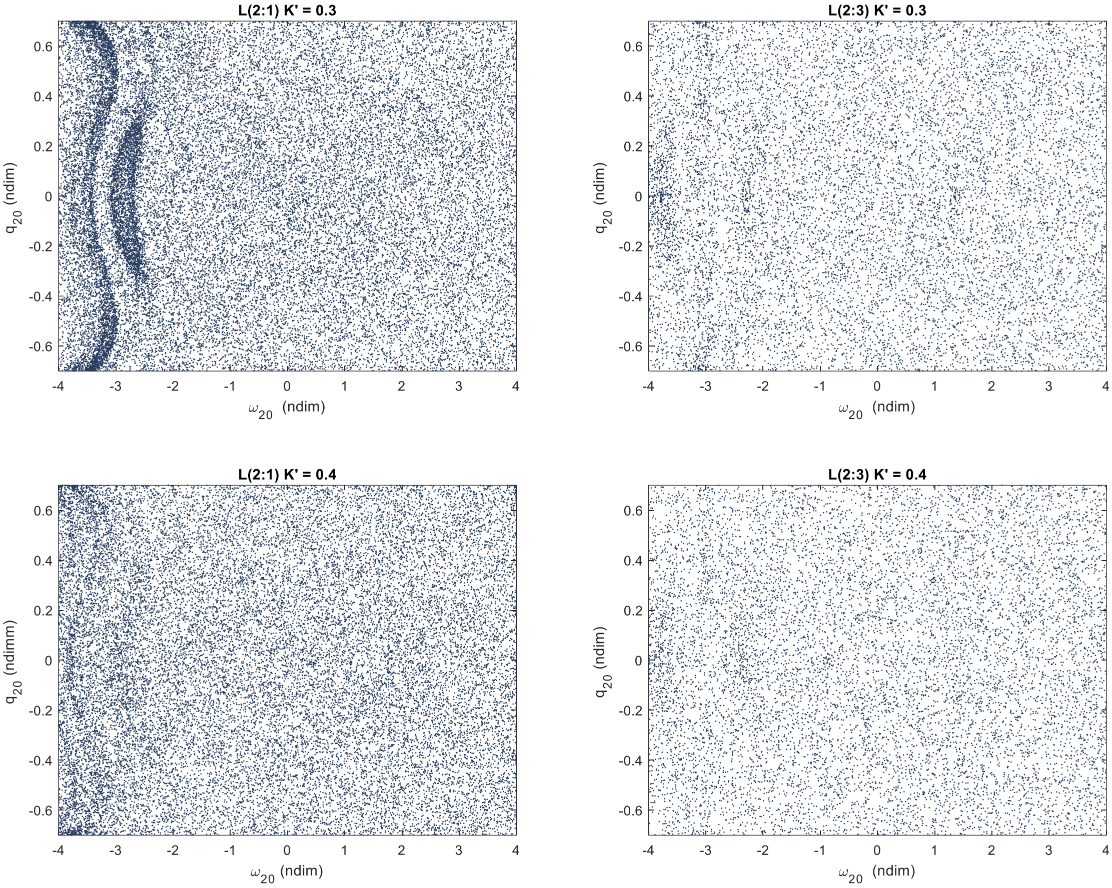


Figure 7. Poincaré mapping for different inertia ratios K for the first two orbits of the Lyapanov family L_2

Conclusion:

In this article, the alternating responses of the orbit-state coupling problem of three circular bounded bodies were studied in the presence of perturbations of both main attractors. For this purpose, in the first step, the equations of orbital motion in the turbulent environment were derived using Lagrangian mechanics. Since the circuit-state coupling equation device does not have an analytical solution, numerical methods were employed to solve it. Considering that these methods can lead to the convergence of the problem to alternating or non-alternating responses according to the initial conditions of the state parameters, and considering the limitation in the number of desired responses, it is necessary to Appropriate initial conditions are used. In order to obtain the initial conditions that lead to circuit-state alternating responses, an algorithm called the circuit-state modification algorithm was developed. The proposed algorithm used the Newton-Raphson multivariate gain approach in order to meet periodic responses. The non-convergence of the proposed algorithm for each desired input indicated the algorithm's need for accurate and appropriate initial guesses. For this purpose, the selection of suitable guesses of orbital state parameters close to the initial conditions of alternating circuits in the mentioned problem was proposed. In the continuation of determining the identification of suitable initial guesses of situational parameters, it was proposed to use Poincaré mapping. Poincaré mapping is effective in finding suitable initial guesses for alternating solutions by considering the desired dynamic flow and its successive passage every time the initial conditions of the flow change from the cross-sectional surface perpendicular to it. The islands obtained in this mapping mean the repetition of similar solutions in the target area, so the guesses that lead to the formation of these islands can be considered as suitable guesses for the solutions. be considered intermittent. The centers of the mentioned islands were considered as initial guesses of the situation parameters. Examining the alternating responses of circuit-state coupling in the mentioned problem revealed that the mentioned responses depend on three basic factors:

The first factor was the reference paths of periodic solutions. The mentioned disturbances with direct effect on these paths will lead to indirect effects on the state parameters. In other words, in the coupling model, considering the environmental disturbances will lead to changes in the reference paths that have visible effects on the state parameters.

The second factor is the identification and selection of the initial guess of the angular velocity ω_{0_2}, which was done using Poincaré mapping. It has also been stated that the use of the angular velocity vector in the form $[0, \omega_{0_2}, 0]$ is very effective in the convergence process of the circuit-state modification algorithm to the desired answers.

The third factor is the selection of appropriate values of the inertia ratio K, which as mentioned, for different values, the problem can have several periodic responses or no periodic response. Identifying the appropriate values of the inertia ratio in this article was done by analyzing the sensitivity of this value on the islands created in the Poincaré map.

One of the most important results of this study was to show the influence of the satellite inertia ratio and the periodicity time of Lyapanovy alternating orbits as a reference path. The results showed that with the increase of the inertia ratio and the alternating time of the reference paths, the number of islands formed in the map formed by the Poincaré map will decrease and the chance of finding alternating state responses will decrease.

Comparing the responses obtained in an environment considering the disturbances of both main absorbers with the environment without disturbances led to a proper understanding of the effect of the mentioned disturbances on the problem of periodic responses of circuit-state coupling. Adding these disturbances by bringing the studied environment closer to the real environment leads to obtaining more accurate answers and a better understanding of the natural motion of the orbit-state of the satellite in the problem of three bounded bodies. It can be a stepping stone in such problems. Be considered.

References:

[1] J. P. Gardner *et al.*, “The james webb space telescope,” *Space Sci. Rev.*, vol. 123, no. 4, pp. 485–606, 2006.

[2] J. Krist, *P. Kalas* “In the Spirit of Bernard Lyot: The Direct Detection of Planets and Circumstellar Disks in the 21st Century, ed,” *(Berkeley, CA Univ. California)*, vol. 32, 2007.

[3] M. Hechler and J. Cobos, “Herschel, Planck and Gaia orbit design,” in *Libration Point Orbits and Applications*, World Scientific, 2003, pp. 115–135.

[4] M. Machula and G. Sandhoo, “Rendezvous and docking for space exploration,” in *1st Space exploration conference: continuing the voyage of discovery*, 2005, p. 2716.

[5] T. R. Kane and D. J. Shippy, “Attitude stability of a spinning unsymmetrical satellite in a circular orbit(Attitude stability of a spinning unsymmetrical satellite, with unequal moments of inertia, in a circular orbit),” *J. Astronaut. Sci.*, vol. 10, pp. 114–119, 1963.

[6] T. R. Kane and E. L. Marsh, “Attitude stability of a symmetric satellite at the equilibrium points in the restricted three-body problem,” *Celest. Mech.*, vol. 4, no. 1, pp. 78–90, 1971.

[7] W. J. Robinson, “Attitude stability of a rigid body placed at an equilibrium point in the restricted problem of three bodies,” *Celest. Mech.*, vol. 10, pp. 17–33, 1974.

[8] B. Wong, R. Patil, and A. Misra, “Attitude dynamics of rigid bodies in the vicinity of the Lagrangian points,” *J. Guid. Control. Dyn.*, vol. 31, no. 1, pp. 252–256, 2008.

[9] M. Sanjurjo-Rivo *et al.*, “On the dynamics of a tethered system near the collinear libration points,” in *AIAA/AAS Astrodynamics Specialist Conference*, 2008, pp. 3218–3242.

[10] D. Guzetti, R. Armellin, and M. Lavagna, “Coupling attitude and orbital motion of extended bodies in the restricted circular 3-body problem: A novel study on effects and possible exploitations,” in *Proceedings of the 63rd International Astronautical Congress, Naples, Italy*, 2012, pp. 5715–5728.

[11] D. Guzzetti and K. C. Howell, “Coupled orbit-attitude dynamics in the three-body problem: A family of orbit-attitude periodic solutions,” in *AIAA/AAS Astrodynamics Specialist Conference*, 2014, p. 4100.

[12] E. Canalias and J. J. Masdemont, “Homoclinic and heteroclinic transfer trajectories between Lyapunov orbits in the Sun-Earth and Earth-Moon systems,” *Discret. Contin. Dyn. Syst*, vol. 14, pp. 261–279, 2006.

[13] J. Singh, A. E. Perdiou, J. M. Gyegwe, and E. A. Perdios, “Periodic solutions around the collinear equilibrium points in the perturbed restricted three-body problem with triaxial and radiating primaries for binary HD 191408, Kruger 60 and HD 155876 systems,” *Appl. Math. Comput.*, vol. 325, pp. 358–374, 2018.

[14] V. K. Srivastava, J. Kumar, P. Mishra, and B. S. Kushvah, “Halo orbit of regularized circular restricted three-body problem with radiation pressure and oblateness,” *J. Astrophys. Astron.*, vol. 39, no. 5, p. 63, 2018, doi: 10.1007/s12036-018-9551-4.

[15] V. V Markellos, K. E. Papadakis, and E. A. Perdios, “Non-linear stability zones around triangular equilibria in the plane circular restricted three-body problem with oblateness,” *Astrophys. Space Sci.*, vol. 245, no. 1, pp. 157–164, 1996.

[16] J. Singh and V. U. Cyril-Okeme, “Perturbed Robe’s circular restricted three-body problem under an oblate

primary," *New Astron.*, vol. 34, pp. 114–119, 2015.

[17] A. E. Perdiou, V. V Markellos, and C. N. Douskos, "THE HILL PROBLEM WITH OBLATE SECONDARY: NUMERICAL EXPLORATION," *Earth. Moon. Planets*, vol. 97, no. 1, pp. 127–145, 2005, doi: 10.1007/s11038-006-9065-y.

[18] A. Colagrossi and M. Lavagna, "Preliminary results on the dynamics of large and flexible space structures in Halo orbits," *Acta Astronaut.*, vol. 134, pp. 355–367, 2017, doi: https://doi.org/10.1016/j.actaastro.2017.02.020.

[19] R. Schwarz, Á. Bazsó, B. Érdi, and B. Funk, "Stability of the Lagrangian point L 4 in the spatial restricted three-body problem–application to exoplanetary systems," *Mon. Not. R. Astron. Soc.*, vol. 427, no. 1, pp. 397–402, 2012.

[20] E. Abbasali, A. Kosari, and M. Bakhtiari, "Effects of oblateness of the primaries on natural periodic orbit-attitude behaviour of satellites in three body problem," *Adv. Sp. Res.*, 2021.

[21] E. Abbasali and M. Bakhteiari, "Restricted three body problem considering the perturbations of both oblate massive primaries," *J. Aerosp. Sci. Technol.*, vol. 13, no. 2.

[22] V. K. Srivastava, J. Kumar, and B. S. Kushvah, "Regularization of circular restricted three-body problem accounting radiation pressure and oblateness," *Astrophys. Space Sci.*, vol. 362, no. 3, p. 49, 2017.

[23] A. E. Ibrahim and A. K. Misra, "Attitude dynamics of a satellite during deployment of large plate-type structures," *J. Guid. Control. Dyn.*, vol. 5, no. 5, pp. 442–447, 1982.

[24] P. HUGHES, "Attitude dynamics of a three-axis stabilized satellite with a large flexible solar array," in *Guidance and Control Conference*, 1972, p. 857.

[25] A. J. Knutson and K. Howell, *Coupled orbit and attitude dynamics for spacecraft composed of multiple bodies in earth-moon halo orbits*, vol. 8. 2012.

[26] K. C. Howell, "Three-dimensional, periodic,'halo'orbits," *Celest. Mech.*, vol. 32, no. 1, pp. 53–71, 1984.

[27] D. Guzzetti and K. C. Howell, "Natural periodic orbit-attitude behaviors for rigid bodies in three-body periodic orbits," *Acta Astronaut.*, vol. 130, pp. 97–113, 2017.

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