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Science Article

Design of Deterministic Self-Tuning Regulators for the Pitch Angle of an Aircraft

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This paper addresses the adaptive control problem of an aircraft and focuses on the task that the pitch angle of the aircraft is required to follow the desired path. Considering the elevator deflection angle as the input and the pitch angle as the output, a mathematical model of the aircraft is derived to specify the structure of the system. Three diverse deterministic self-tuning regulators are designed using direct and indirect methods. Assuming that the system is unknown, recursive least squares method is applied to estimate parameters of the system or that of the controller's. Diophantine equation and minimum degree pole-placement methods are utilized to calculate the control law. Not only do simulation results clearly demonstrate the privilege and effectiveness of the proposed approaches, but also comprehensive discussion is presented to distinguish advantages and disadvantages of them.

Keywords: *"Aircraft", "Pitch Angle", "Adaptive Control", "System Identification", "Self-tuning Regulators".*

Introduction

Undoubtedly, aerospace engineering was the birthplace for adaptive control. In the course of world war two and afterwards, designing autopilots for high-performance aircraft was one of the fundamental motivations for active research on adaptive control in the early 1950s. Using developed theories of adaptive control and combining them with system identification methods, in this paper we are going to deal with pitch angle control problem of an aircraft.

Generally speaking, in flight, any aircraft will rotate about its center of gravity, a point which is the mean location of the mass of the aircraft. A three dimensional coordinate system can be defined through the center of gravity with each axis of this coordinate system perpendicular to the other two axes. Then the aircraft orientation can be defined by the amount of rotation of the parts of the aircraft along these principal axes. The pitch axis is perpendicular to the aircraft centerline and lies in the plane of the wings. A pitch motion is an up or down movement of the nose of the aircraft. The pitching motion is being caused by the deflection of the elevator of this aircraft. Changing the angle of deflection at the rear of an airfoil changes the amount of lift induced by the foil. With greater downward deflection, lift increases in the upward direction. With greater upward deflection, lift increases in the downward direction. The lift generated by the elevator acts

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through the center of pressure of the elevator and horizontal stabilizer and is located at some distance from the center of gravity of the aircraft. The change in lift created by deflecting the elevator generates a torque about the center of gravity which causes the airplane to rotate.

The literature of aircraft control is incredibly rich. To mention some of closest works to ours, we may refer to [1], [2], [3] and [4]. In [1], Cadwell's dissertation concentration was to minimize pitch axis gain variation as center of gravity changes. Based on Ziegler-Nichols closed loop tuning methods, he designed a PD (proportional-derivative) controller to stabilize and minimize gain variance of an aircraft flight model. In [2], adaptive control of the aircraft pitch angular motion by using the dynamic inversion principle was discussed.

Eventually, we are going to merge system identification methods , to estimate parameters of an aircraft pitch angle transfer function, with control theories and design self-tuning regulators in order to track the given pitch angle reference path.

Mathematical Modeling

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is governed by the longitudinal dynamics. In this paper we will design self-tuning regulators that control the pitch of an aircraft.

Fig. 1 shows basic coordinate axes and forces acting on an aircraft.

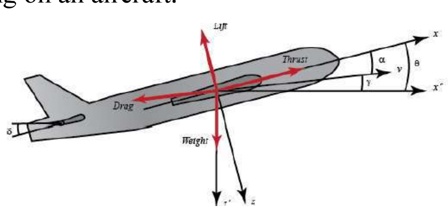

Fig. 1: Basic coordinates and forces acting on an aircraft

We will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-directions. We will also assume that a change in pitch angle will not change the speed of the aircraft under any circumstance. Under these assumptions, the longitudinal equations of motion for the aircraft can be written as follows.

$$
\begin{aligned}
\dot{\alpha} &= \mu\Omega\sigma[-(C_T + C_D)\alpha + \frac{1}{(\mu - C_L)} q - (C_W \sin\gamma)\theta + C_L] \\
\dot{q} &= \frac{\mu\Omega}{2 i_{yy}}[[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L) q + (\eta\, C_W \sin\gamma)\delta] \\
\dot{\theta} &= \Omega\, q
\end{aligned} \tag{1}
$$

In which α is the angle of attack, q and θ are pitch rate and pitch angle respectively, δ is the elevator deflection angle, μ equals $\frac{\rho s \bar{c}}{4m}$ with where ρ, s and m are density of air, platform area of the wing and average chord length respectively, Ω equals with $\frac{2u}{\bar{c}}$ where U is the equilibrium flight speed, $C_T C_D C_L C_W$ and C_M are coefficients of thrust, drag, lift, weight and pitch moment respectively, γ is the flight path angle, σ equals with $\frac{1}{1+\mu c_L}$ and is constant, is i_{yy} the normalized moment of inertia and eventually equals η with $\mu\sigma c_L$ and is constant. Using some numerical values from Boeing's commercial aircraft we will have:

$$
\begin{aligned}
\dot{\alpha} &= -0.313\,\alpha + 56.7\,q + 0.232\,\delta \\
\dot{q} &= -0.0139\,\alpha - 0.426\,q + 0.0203\,\delta \\
\dot{\theta} &= 56.7\,q
\end{aligned} \tag{2}
$$

By defining the elevator deflection angle δ as the input and the pitch angle θ as the output and taking Laplace transform from above equations, considering zero initial condition, the system continuous transfer function will be derived as follows:

$$
G(s) = \frac{\theta(s)}{\Delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} \tag{3}
$$

Which has the following root locus diagram:

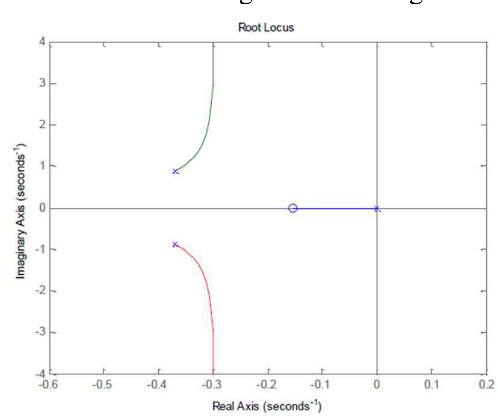

Fig. 2: Root Locus diagram of the system

Using zero order hold method and a suitable sampling time, $T_s = 0.2$, the discrete system will be derived as follows:

$$G_d(z) = \frac{0.02209z^2 + 0.0001757z - 0.0206}{z^3 - 2.828\, z^2 + 2.691\, z - 0.8626} \quad (4)$$

$$G_d(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{0.02209\, z^{-1} + 0.0001757z^{-2} - 0.0206z^{-3}}{1 - 2.828\, z^{-1} + 2.691\, z^{-2} - 0.8626z^{-3}} \quad (5)$$

Needless to mention that sampling time was determined as though the continuous and discrete systems show an identical behavior.

In the following sections we are going to consider this discrete system and assume that the parameters are unknown. We will use system identification methods and combine them with pole placement algorithms to design controllers which meet desirable performance requirements.

System Identification in Adaptive Control

Any adaptive controller is formed from two stages that work simultaneously in an online manner, system identification and control law. In the identification part recursive least squares (RLS) method is used to estimate parameters.

$$\begin{gathered} P(t) = p(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1+\varphi^T(t)P(t-1)\varphi(t)} \\ \varepsilon(t) = y(t) - \varphi^T(t)\hat{\theta}(t-1) \\ \hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\varphi(t)\,\varepsilon(t) \\ \theta(0) = 0 \quad P(0) = \alpha I \quad \alpha > 0 \end{gathered} \quad (6)$$

In the above RLS equations,P is kind of the covariance matrix and ε is the one-step ahead prediction error. θ is the parameters vector and φ is the regressors vector. Since the equations are online and recursive, an appropriate initialization is needed for convergence. I is the unity matrix and α is a very large constant, usually chosen between 10^4 and 10^6.

In the second stage, pole placement algorithms will be utilized to design well functioning controllers, exploiting obtained information from the identification part.

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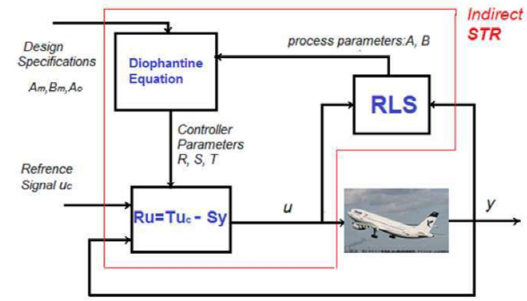

Fig. 3: Indirect STR diagram

In indirect adaptive control, the plant parameters are estimated online and used to calculate the controller parameters. Assuming the fact that the system has the structure of $G_d(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$ and if the ideal closed-loop system behavior that includes desirable poles and zeros is denoted like $A_m Y_m(t) = B_m u_c(t)$ and defining $B = B^+B^-$ where B^+ is a monic stable polynomial with well damped roots and B^- is unstable or poorly damped roots polynomial, for perfect model following it is necessary to have $B_m = B^- B_m{}'$ Eventually, controller will have the form of

$$R_u = Tu_c - sy \quad (7)$$

In which u is the control signal, u_c is the reference signal, y is the output of the system and R, T and S are polynomials which are obtained with following step by step procedure: The subsequent Diophantine equation yields polynomials R' and S:

$$AR' + B^- S = A_o A_m \quad (8)$$

Then polynomials R and T are computed as follows:

$$R = R'B^+ \quad (9)$$

$$T = A_o B_m{}' \quad (10)$$

On contrary with indirect methods, in direct adaptive control, the plant model is parameterized in terms of the controller parameters that are estimated directly without intermediate calculations involving plant parameter estimates.

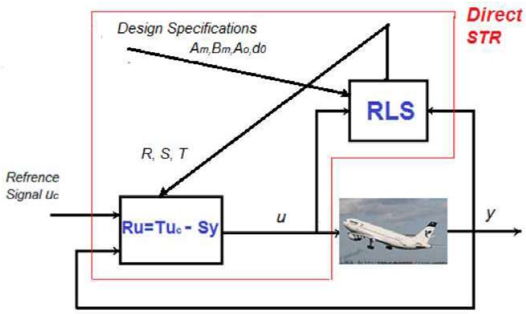

Fig. 4: Direct STR diagram

Comparing figures 3 and 4, it is evident that the controller design block has vanished in direct STR. So it is anticipated that computation complexity reduces in direct STR rather than the indirect one.

Indirect STR for the Pitch Angle of an Aircraft

In this section steps of designing a self-tuning regulator for the pitch angle of an air craft are discussed.

We assume that the system in unknown for us and structure of the model - number of poles and zeros- is the only priori knowledge we have about the system.

Therefore, we apply RLS to identify parameters of the system, that is, considering numerator and denominator of the system having the forms of $A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3}$ and $(z^{-1}) = b_0z^{-1} + b_1z^{-2} + b_2z^{-3}$ aim of identification part is to estimate a_i and b_i .

Figures 5 and 6 show the estimation of parameters that converge to their actual values. The red dashed lines are the actual values and blue curves are the estimated parameters in simulation span.

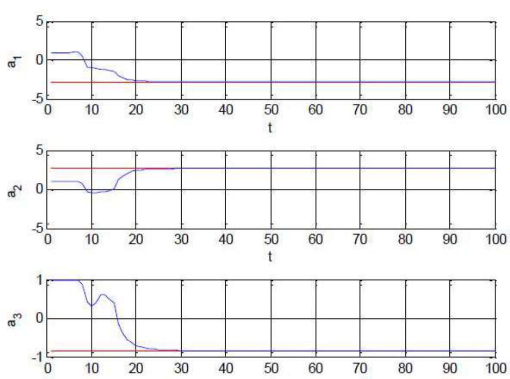

Fig. 5: Denominator coefficients estimation (Indirect STR)

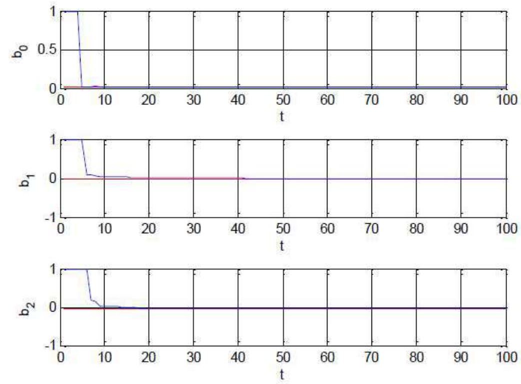

Fig. 6: Numerator coefficients estimation (Indirect STR)

It is noteworthy to mention that actual values of b_0, b_1 and b_3 are 0.02209 , 0.0001757 and 0.0206 respectively, which are really close to zero.

Now that a model of the system is obtained, using this model and applying the pole placement with pole zero cancellation algorithm (discussed in previous section and thoroughly in [6] and [7]), a self-tuning regulator is going to be designed to track the given reference signal u_c. reference signal u_c is a step function altering between 1 and -1.

After some algebraic calculations as was discussed in the preceding section, the control law is computed as follows:

$$u(t) = 9.5024\, u_c(t) + 0.0089\, u(t-1) + 0.9320\, u(t-2) - 73.6797y(t) + 100.4871y(t-1) - 36.3097y(t-2) \quad (11)$$

As can been seen in Fig.7 (a), output of the system perfectly tracks the reference signal. In the course of first step pulse tracking, overshoot is less than 25% which is acceptable. Afterwards, when the reference signal switches, output of the system tracks the desired path even without overshoot. The reason lies in the fact that during the first step pulse, parameters of the system had to be identified and this identification process caused a larger overshoot. Moreover, since no fluctuation is seen in the transient response and all the oscillations damp rapidly, the settling time stays very close to the rise time, therefore the speed of the system is relatively acceptable.

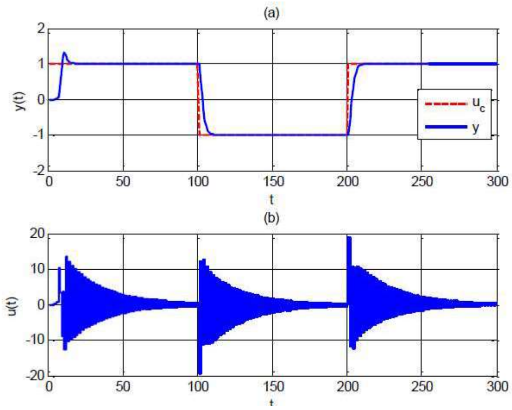

Fig.7: Indirect STR with Pole-Zero Cancellation: (a) Tracking (b) Control Signal

While it was taken into account that control signal stays in a feasible region, the only adverse observation here is the so called *ringing* phenomenon. That is, the control signal fluctuates rapidly in transient response. Although it was anticipated, due to the fact that pole-zero cancelation method was utilized, but this *ringing* phenomenon may cause some implementation problems if actuators cannot handle it.

Direct STR for the Pitch Angle of an Aircraft

In direct self-tuning regulators algorithm, coefficients of polynomials R and S are estimated directly. Therefore, the complexity of computation is lower than the indirect method. Anyhow, the control signal will have the form of:

$$u(t) = 21.7195\, u_c(t) + 0.009\, u(t-1) + 0.9321\, u(t-2) - 100.8351y(t) + 118.1457y(t-1) - 39.03y(t-2) \qquad (12)$$

Fig.8 (a) shows the tracking result of the direct method.

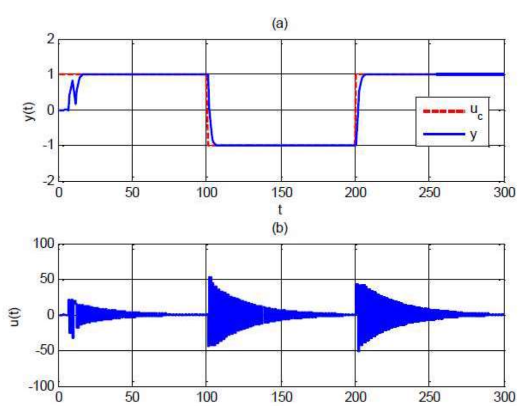

Fig.8: Direct STR with pole-zero Cancellation: (a) Tracking (b) Control signal

A fascinating fact about direct self-tuning regulators is that parameters may not converge to their actual values while the tracking takes place perfectly.

Again, the so-called *ringing* phenomenon is observed in the control signal because of pole-zero cancellation. This fluctuating of the control signal is what is paid at the cost of lower computational complexity and higher speed.

By the way, Fig.9 shows the error between the reference model and the system outputs. By reference model we mean our desirable behavior for the closed loop system which was donated by $A_m Y_m(t) = B_m u_c(t)$

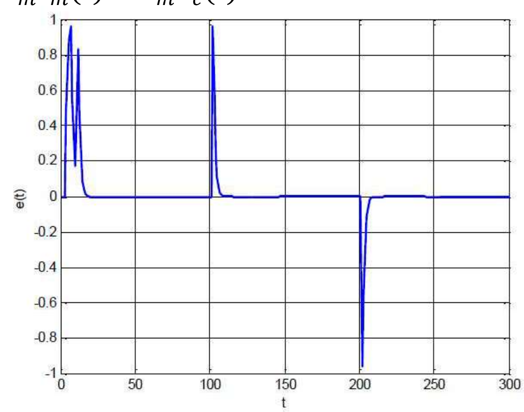

Fig. 9: Error signal $e(t) = y\_m(t) - y(t)$ (Direct STR with pole-zero cancellation)

To distinguish the discrepancy between algorithms with pole-zero cancellation and without pole-zero cancellation, another indirect simulation was performed for this very aircraft system, this time without pole-zero cancellation. Figures 10 and 11 show the parameters convergence and result of model following respectively.

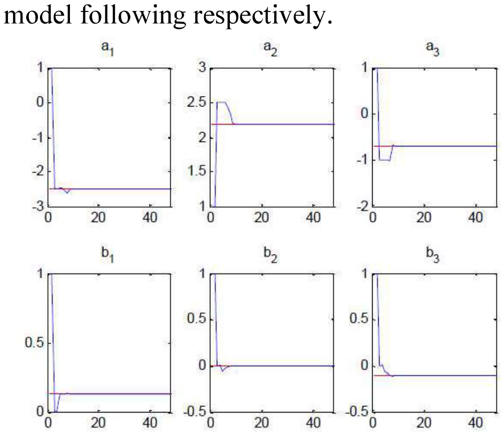

Fig. 10: Parameters estimation (Indirect STR without pole zero cancellation)

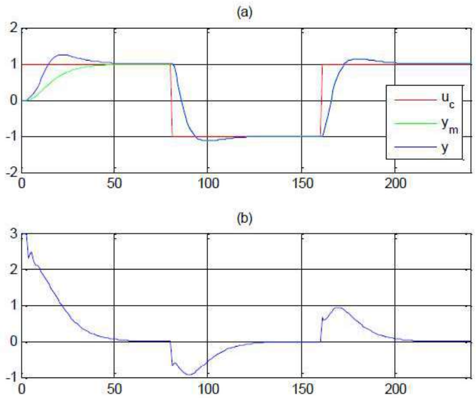

Fig. 11: Indirect STR without pole-zero cancellation (a): Tracking (b): Control signal

As can be seen, the control signal happened to be way too smoother while the speed of the system has diminished considerably.

So to put the whole thing in a nutshell, pole-zero cancellation method causes the ringing phenomenon which is good from identification point of view but may not be desirable from practical point of view. Moreover, the computation complexity is less consequently the response is faster. However, without pole-zero cancellation method is somehow slower but the control signal is very smooth and pleasant.

Concluding Remarks

Combining online system identification methods (RLS) with minimum degree pole-placement algorithms, three deterministic self-tuning regulators were designed for the pitch angle of an aircraft which showed promising results in simulations.

By analyzing the results, it can be observed that in indirect STR with pole-zero cancellation the parameters convergence to their actual values take place relatively fast but control signal may face the *ringing* phenomenon which may causes some difficulties in implementation. It is noteworthy to mention that this *ringing* phenomenon is desirable from the identification standpoint since it makes the signal more persistent exciting. However, in indirect STR without pole-zero cancellation, convergence is more time consuming and system is relatively a little bit slow but control signal is absolutely smooth.

Therefore, considering the capacity of actuators, there always exists a subtle trade of between the speed of the system and the quality of the control signal.

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