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Science Article

Optimal Guidance of a Reentry Vehicle Based on Online Trajectory Optimization

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In this paper, a new method for optimal guidance in the atmospheric return phase is proposed. This guidance method is based on instantaneous and online trajectory optimization in which optimal guidance commands are obtained from sequential solving of optimal control problems. In order to solve optimal control problems quickly and online, a combined approach including the concepts of differential flatness, B-spline curves, direct collocation, and non-linear programming is used. By performing the trajectory optimization process in the form of closed-loop control and implementing the receding horizon control, the open-loop responses of optimal control can be dependent on the instantaneous conditions of the object and the target. In this case, guidance commands can be generated based on various objective functions and constraints, and model uncertainties can be considered by entering the vehicle conditions into the trajectory optimizer. In order to show the capabilities of the proposed guidance method, a numerical example of the guidance of a reentry vehicle in the presence of model uncertainties is presented.

Keywords: *optimal guidance of reentry, online trajectory optimization, differential flatness, B-spline curves, receding horizon control*

Introduction

Guidance of spacecraft in the Earth's atmospheric return phase is a major challenge for designers of optimal trajectories and guidance laws. The structural limitations of the vehicles and their mission requirements cause a great deal of sensitivity to the accuracy of the guidance laws. In vehicles such as research spacecraft and bio capsules, the return trajectory should be such that a smooth and slow landing of objects is achieved on the ground, and in vehicles such as warheads, the return trajectory should be such that the vehicle is at maximum velocity and hits the target in the shortest time. Hence, the nature of the vehicles and their missions outline the goals of the guidance process for designers. In the last half-century, various guidance laws have been proposed for the guidance of aerospace vehicles, which are typically used to guide the vehicle in the atmospheric return phase. Early and classical guidance laws are generally based on the geometry and kinematics of vehicles and targets. Laws such as pursuit, proportional navigation, and other types derived from these laws (such as PPN, TPN, and IPN) fall into this category [1]. The output of these laws is the acceleration commands of the vehicle that can guide the vehicle to the target. New control laws are generally derived from the theory of optimal control. By using optimal control, it is possible to achieve optimal guidance of vehicles. Optimal control can also be used for demonstrating the optimality of some classical guidance laws [٢].

In 1965, for the first time, the problem of optimal flight of spacecraft in the atmospheric return phase was solved using optimal control [3]. Of the significant work that has been done in recent years in this regard, we can refer to the work carried out

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by Eisler and Hull [4-6]. In [4], the researchers were able to use the feedback control method of the sampled data to maximize the impact velocity of the vehicle to hit a specified target and achieve a near-optimal response. In [۶,5], using the inverse method and Bezier curve, an explicit guidance law is developed for the problem raised in [4]. In order to achieve analytical solutions, most of the new guidance laws are derived from the analytical solution of optimal control problems. In these laws, analytical solutions are obtained based on various hypotheses and simplifications. By eliminating these simplifications and defining the guidance problem in its most general form in three-dimensional space and with a non-linear dynamic model, a complex non-linear optimal control problem is obtained that has no analytical solution. The numerical solution of this problem is also an open-loop that is not directly dependent on the state variables and, unlike the classical guidance laws, is not a closed-loop and feedback response. Therefore, we should look for a structure that makes the generated solutions of the optimal control problem dependent on the state variables and make the guidance structure closed-loop. Successful implementation of such an approach requires a very fast and accurate solving of the problem of optimal control. If the problem can be solved online in a short period of time, the optimal control commands can be applied to the vehicle, and the vehicle state variables can be fed back to the optimal control problem solver. This closed-loop structure can be implemented in the form of methods such as predictive model control (receding horizon control).

In this paper, we want to define the problem of an optimal guidance of a warhead in the atmospheric return phase and solve it with a combined method. In this method, by combining different concepts such as differential flatness, B-spline curves, direct collocation, and non-linear programming, a new approach is used to quickly, accurately, and online solve optimal control problems. In the combined method, the optimal guidance problem is formulated based on the concept of differential flatness to define the problem in the minimum dimensional space and with the minimum number of variables and constraints. Then, the variables used are approximated by B-spline curves to express continuous variables over time with discrete values. For applying point and path constraints in this approach, the concept of direct collocation is used. That is, constraints are applied and satisfied at certain points in time called nodes. After performing these steps, the discrete problem of optimization is obtained, which can be solved by conventional non-linear programming methods. By creating a closed-form control loop based on the principles of receding horizon control and considering the trajectory optimizer section as a non-linear controller, the instantaneous conditions of the vehicle and the target can be entered into the optimizer section, and optimal guidance commands can be obtained for a specific time horizon and applied in a small part of the time horizon. By repeating this procedure and solving the problem of optimal guidance several times, it is possible to generate guidance commands online and based on the instantaneous conditions of the vehicle and the target.

Definition of the optimal guidance problem in the atmospheric return phase

In recent decades, the issue of spacecraft optimal flight in the phase of returning to the Earth's atmosphere has been considered by various researchers around the world, and various approaches have been proposed to formulate and solve this problem. One of the important views in defining this issue is its expression in the form of an optimal control problem. Using the concepts of optimal control theory, the problem can be defined and solved with acceptable accuracy, and at the same time, conventional simplifications cannot be used to solve guidance problems. That is, the problem can be defined in a more complex form, and a closer solution to reality can be obtained for it. According to work done in this field, the general form of the guidance problem based on optimal control can be assumed as follows:

"After returning to the atmosphere, we want to determine the flight trajectory of a spacecraft to optimal reach to a target such that the vehicle satisfies the trajectory constraints (structural and mission constraints) despite model uncertainties and perturbations."

To express such a problem mathematically, we must first determine the motion equations of the vehicle in the state space and then define the optimal control problem based on it. By determining the motion equations, the state and control variables are also specified, and the trajectory constraints, point constraints, and objective function can be defined according to the problem situation. In the present paper, to express the dynamic model of the guidance problem, we use two-dimensional motion equations of the vehicle in the inertial coordinate system.

Due to the increasing costs and environmental challenges that lie ahead of increasing the planes; efficiencies, plane manufacturers have been under a lot of pressure. One of the main factors that contributes to this problem is the high price of fuel, the need for lower pollutants and the demand for creating environmental-friendly airplanes that help to lower the effects of global warming [1].in the field of aerospace engineering, drag reduction poses a great and challenge for engineers, so it could still be improved and novel ideas and creative works could still change the fate of this field [2]. The flow over the wing of an airplane is a three-dimensional; that is, a factor of the flow is aligned with the wingspan. the difference in pressure distribution results into the creation of lift force. In addition, this pressure differential between upside and downside of the wings, transfers the high-pressured flow which is below the wings to the above-surface of the upper wing, creating a vortex on both wing-edges of the airplane [3]. In aviation and aerospace, the existence of such vortices is dangerous and causes air traffic for the airports. These vortices are so strong that it takes at least 2 minutes for them to weaken and shed. This two-minute time is actually the time gap between two landings and takeoffs in an airplane [4]. These vortices finally lead to the induced drag force, which its reduction is one of the primary goals. reducing the drag can be beneficial and reduce fuel-consumption and increase the flight range. In fact, these environmental challenges and functional costs have impelled the aviation industry into finding new ways to increase the efficiency and frugality of commercial air transportation, which in turn have resulted into emergence of some novel and innovative ways for reducing induced drag [5].

$$\dot{V}_x = -L\sin(\gamma-\beta)/M - D\cos(\gamma-\beta)/M - g\sin\beta \tag{1}$$

$$\dot{V}_y = +L\cos(\gamma-\beta)/M - D\sin(\gamma-\beta)/M - g\cos\beta \tag{2}$$

$$\dot{x} = V_x \tag{3}$$

$$\dot{y} = V_y \tag{4}$$

Where x and y indicate the position of the vehicle, and V_x and V_y represent the velocity components of the vehicle in the inertial coordinate system.

According to Figure 1, the β and γ angles can be calculated as follows:

$$\beta = \tan^{-1}\left(\frac{x}{y}\right) \tag{5}$$

$$\gamma = \tan^{-1}\left(\frac{V_x}{V_y}\right) + \beta \tag{6}$$

Where M is the mass of the vehicle and g is the gravity acceleration, which can be calculated using the following equations:

$$R = \sqrt{x^2 + y^2} \tag{7}$$

$$g = 9.81\left(\frac{R_E}{R}\right)^2 \tag{8}$$

Where R is the distance of the vehicle from the center of the Earth and R_E is the average radius of the Earth. Lift (L), drag (D), and weight (M_g) are the forces acting on the vehicle, as shown in Figure 2.

The following equations can be used to calculate the values of lift and drag forces:

$$L = 0.5\rho V^2 S C_L \tag{9}$$

$$D = 0.5\rho V^2 S C_D \tag{10}$$

Where S is the reference surface, ρ air density, and V is the velocity of the vehicle. The velocity and height (H) of the vehicle can be calculated using the following equations:

$$V = \sqrt{V_x^2 + V_y^2} \tag{11}$$

$$H = R - R_E \tag{12}$$

By having the vehicle height and standard atmospheric model, air density and velocity of sound (c) can be determined. By using the velocity of sound and the velocity of the vehicle, the *Mach* number can be calculated:

$$Mach = V / c \tag{13}$$

Lift (C_L) and drag(C_D) coefficients are calculated from the following equations:

$$C_L = C_{L\alpha}\alpha \tag{14}$$

$$C_D = C_{D0} + C_{D\alpha}\alpha^2 \tag{15}$$

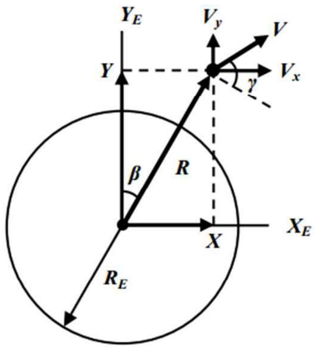

Figure 1- Definition of variables in the inertial coordinate system

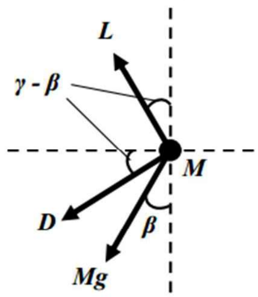

Figure 2- Applied forces on the vehicle

The aerodynamic coefficients can be calculated using Mach number and in accordance with the aerodynamic model of the vehicle. In these equations, α is the attack angle of the vehicle, which is the control variable of the equations of motion. By determining the time history of α, the vehicle can be guided to various targets. In the stated dynamic model, there are four state variables (x, y, V_x, V_y) and a control variable (α). In a guidance problem, the main objective is reaching the vehicle to the target. Accordingly, in addition to the initial conditions of the vehicle, the final conditions of the vehicle are also specified at the end of the mission. In addition to this main objective, other objectives can also be considered. For example, maximizing collision velocity, maximizing longitudinal range, minimizing time, and so on can be considered mission objectives. In a guidance problem, various constraints and limitations can also be defined. For example, restrictions can be considered for control variables or structural and mission considerations of the vehicle, which can be expressed in the form of constraints. By determining the angle of attack during the mission time, the vehicle can be aimed at the target. If the angle of attack is considered zero, i.e., the vehicle moves without applying control commands, the nominal trajectory of the vehicle is obtained. For this purpose, we consider the following basic conditions:

$H_{nom} = 90$ km , $V_{nom} = 3600$ m/s , $\gamma_{nom} = -30°$, $\beta_{nom} = 0°$

By considering the above values, the initial values of the state variables can be obtained as follows:

$V_x = 3117.69$ m/s , $V_y = -1800$ m/s , $x = 0$ km , $y = 6460$ km

By solving the motion equations with the above initial conditions, the nominal trajectory of the vehicle is obtained, which is shown in Figure 3. Figure 4 also shows the changes in state variables with time. The collision point of the vehicle with the ground is obtained in the coordinates x = 140.80 km and y = 6368.44 km. If we consider the vehicle's collision with this point as the target of guidance, we must look for the law that can guide the vehicle to the specified target despite the uncertainties of the model and the perturbations.

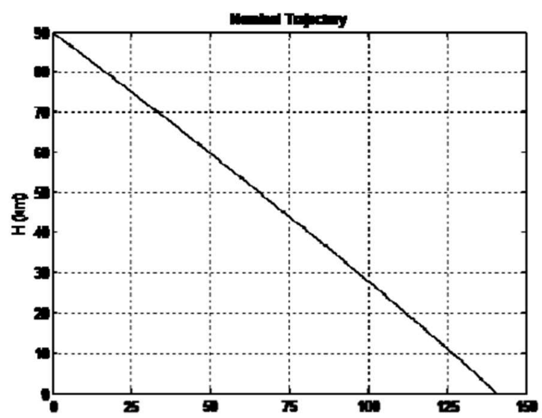

Figure 3- The nominal trajectory of the vehicle

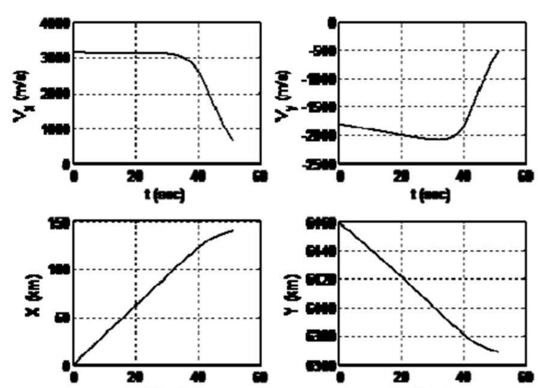

Figure 4- Variation of state variables with respect to time

An online combined method for solving optimal control problems:

In general, there are three common methods for solving optimal control problems, which are: the indirect method, direct shooting method, and direct collocation method.

Other solution methods are somehow developed or a subset of these three methods. Among these methods, the direct collocation method has a higher solution velocity due to its completely numerical solution structure. In this method, by completely discretizing the problem and converting differential and integral equations to simple algebraic equations, it is possible to use non-linear programming methods. This method leads to a discrete solution and, due to the production of a large number of variables and optimization constraints in it, cannot be implemented online. [7,8] In recent years, a combined method has been proposed to develop a direct collocation method that allows it to be used online. [9] The combined method for online trajectory optimization is based on the simultaneous use of the concepts of differential flatness, B-spline curves, direct collocation, and non-linear programming. In this approach, by using the concept of differential flatness, the dimensional space of the trajectory optimization problem is reduced, and the problem is expressed with the minimum number of variables and state equations. Also, by using B-spline curves, despite maintaining the discrete nature of the optimization variables, a continuous concept of the solution is obtained, and the role of time nodes in approximating the differential and integral expressions of the problem is eliminated. In this approach, by using the concept of collocation and time nodes, the path and point constraints are applied in time nodes. Finally, the control points of B-spline curves are considered non-linear programming optimization variables. Due to the very fast solution of the trajectory optimization problem with the mentioned approach, it is possible to implement it online in the form of guidance and control loops.

In the following, each component of the combined method will be discussed in detail, and the role of these components in providing the possibility of online trajectory optimization will be explained.

In the classical direct collocation method, the collocation process is applied to all state and control variables. This causes a large number of optimization variables and constraints to be generated. In recent years, a group of researchers has shown that by removing control variables from state equations, the collocation process can be applied only to state variables, and after convergence and problem-solving, using the optimal values calculated for state variables, the optimal values of control variables can be calculated. [10] This approach is called the inverse method. In the inverse method, first, using state equations, the existing equation between control variables and their first-order derivatives of state variables and time are obtained. Then, the remainder of the equations of state is rewritten according to the obtained equation. In this method, the finite difference method is used to calculate the derivatives of state variables. The advantage of the inverse method is the removal of control variables from the collocation, convergence, and solution processes. In this method, the state equations that are used to obtain equations between the state and control variables are also eliminated. Using this method is somewhat effective in accelerating the solution. However, in recent years, another group of researchers has shown that in addition to control variables, some state variables can be eliminated from the problem, and the collocation process can be applied only to the remaining state variables. After solving, the optimal values of the eliminated state and control variables can be calculated using the existing equations. The latter approach is only possible in differentially flat non-linear systems. A non-linear dynamic system is differentially flat when there is a variable change for it as follows[10]:

$$\mathbf{q} = \mathbf{h}(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}, \ldots) \tag{16}$$

So that the state (x) and control (u) variables can be obtained as follows:

$$(\mathbf{x}, \mathbf{u}) = \mathbf{w}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \ldots) \tag{17}$$

The variables **q** can be equivalent to some state variables or a combination of them. These variables are not necessarily sensor-readable variables and are known as flat outputs.

A remarkable feature of flat systems is that the whole behavior of the system can be expressed without integration and only using flat outputs and a limited number of their time derivatives. However, it should be noted that due to the use of derivatives of flat outputs, using the finite difference method for calculating derivatives will no longer be reasonable.

If the concept of differential flatness is used in trajectory optimization, the collocation and approximation process can be applied only to flat outputs instead of all state and control variables. By doing this, a drastic reduction in the number of optimization variables occurs. Also, due to the use of state equations to obtain the equations between flat outputs and other state variables and problem control, a number of state equations are practically eliminated, which greatly reduces the number of optimization constraints.

An interesting advantage of using the concept of differential flatness in trajectory optimization is that the number of variables and equations are reduced together. This simultaneous reduction causes a significant reduction in the Hessian of Lagrangian function matrix. According to the time-consuming calculations of these matrices (about 70% of the solution time), this reduction has a significant effect on increasing the solution speed.

The lack of a systematic method for detecting differential flatness of a system, as well as the lack of a suitable algorithm for determining the minimum flat outputs, make it difficult to use differential flatness to optimize the trajectory. It should be noted that in the combined approach, we use only the concept of differential flatness. In this approach, the flatness of the system is not important. That is, even if the system was non-flat, we still identify the flat outputs and obtain other state and control variables based on the flat outputs. The only difference here is that if the system is non-flat, we will have equations of state in the reduced space of the problem. Therefore, to use the concept of differential flatness in trajectory optimization, there is no need to distinguish between differentially flat and non-flat systems.

In conventional dynamical systems, the determination of flat outputs by trial and error is not very complicated. Positional variables are usually the best choice for flat outputs. Because by having the time function of positional variables, the values of other state and control variables can be obtained at different times. In fact, if the physical trajectory taken by a vehicle is known, its state and control variables can be calculated. Therefore, to use the concept of differential flatness in trajectory optimization, there is no need to develop a special algorithm to determine flat outputs.

In the direct collocation method, the state and control variables are approximated as a set of discrete points. This causes the nature of the solution to be discrete, and it is not possible to calculate the derivative and integral of the functions accurately. This problem becomes even more acute if the concept of differential flatness is used because, in that case, it is not possible to calculate the higher derivatives with discrete points accurately. This discretization converts the problem of trajectory optimization to a non-linear programming problem, and it is impossible to ignore it. One solution to this problem is to use curves. In curves, although coefficients or control points are discrete values, they generate a continuous concept. If the state and control variables are approximated to the curves, the problem discreteness is maintained due to the discontinuity of coefficients or control points, and it is also possible to calculate derivatives and integrals accurately due to having the curve functions. In the combined method, we use B-spline curves to approximate the variables of the trajectory optimization problem. These curves are an interconnected set of Bezier curves[11]:

$$x(t) = \sum_{i=0}^{n} B_{i,k}(t) C_i \qquad t_0 \le t \le t_f \tag{18}$$

The equations of B-spline curves consist of two parts: basis functions ($B_{i,k}(t)$) and control points (C_i). In order to calculate the basis functions of a B-spline curve, the number of Bezier curves and their degrees must be determined in proportion to the complexity of the expected trajectory for the approximated variable. Then, the time period should be divided according to the number of Bezier curves. The time points are called nodes. By placing the time values of the nodes in a vector, the node vector (τ) is formed:

$$\boldsymbol{\tau} = [t_0, t_1, \ldots, t_{n\text{-}1}, t_n] \tag{19}$$

In a node vector, a value of time may be repeated several times in a row, which is called the number of repetitions. The difference between the order of the curve (k_i) and the number of corresponding nodes (m_i) determines the degree of smoothness (s_i):

$$s_i = k_i - m_i \tag{20}$$

The value of smoothness indicates the level of continuity in the node, which is equal to the order of derivation. Therefore, the level of

continuity in nodes can be applied in the definition of the node vector by repeating the time values of the nodes. It should be noted that the order of the curve is equal to the value of the degree of the curve plus one.
By having the order of the curves (k), the number of the curves (l), and the smoothness of the nodes (s), the number of control points (p) can be determined.

$$P = l(k - s) + s \tag{21}$$

Once the above cases are known, the basis functions can be calculated using the following equations:

$$B_{i,0}(t) = \begin{cases} 1 & \text{if } t_i \le t \le t_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$B_{i,k}(t) = \frac{t - t_i}{t_{i+k+1} - t_i} B_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(t) \tag{23}$$

The control points in a B-spline curve are the points around the curve that form the curve and literally control the curve. The control points in B-spline curves, such as polynomial coefficients, are discrete values that generate a continuous concept. In approximating the state and control variables, these control points can be considered optimization variables. B-spline curves behave quite locally. By changing one of the control points, depending on the degree of the curve, only the shape of the curve in the vicinity of the control point changes, and the rest of the curve remains unchanged. Also, the range of changes of control points in B-spline curves is the same and is approximately equal to the range of changes of the approximated variable. If B-spline curves are used to approximate the variables of the trajectory optimization problem, the accurate calculation of the time derivatives is easily possible due to the specificity of the time derivatives of B-spline curve functions. B-spline curves, due to the optimal approximation of complex trajectories, discreteness of control points, expression of a continuous concept, no need to define continuity constraints, completely local behavior, and the same range of changes of control values with approximated variables, are very suitable curves for approximating state variables and control in trajectory optimization.
In the direct collocation method, time nodes are used for two purposes. The first purpose is to approximate differential and integral expressions. Hence, to achieve better approximations, a large number of time nodes must be selected. Another purpose is to apply constraints to the trajectory optimization problem [1۳]. If B-spline curves are used, the role of time nodes in the approximation of differential and integral expressions is eliminated, and time nodes are used only to apply the constraints of the problem. This eliminates the need to use a large number of time nodes.
For solving the problem of trajectory optimization by the combined method, the flat outputs determined from the differential flatness are approximated by B-spline curves. By doing this, the control points of the mentioned curves are considered optimization variables of the non-linear programming problem. During the solution process, depending on the value of the optimization variables in each iteration, the control points of the B-spline curves are specified. By knowing these points and consequently the functions of the curves, the values of other state and control variables can be calculated. By applying the remaining state equations to time nodes (collocation points), virtually all state equations are applied to the problem. In addition to the residual state equations, trajectory constraints are applied to time nodes. After convergence and solving the non-linear programming problem, the optimal values of the control points are obtained in such a way that the state equations and the trajectory constraints are satisfied in the time nodes, and the objective function is minimal. In order to scale, bound, and generate optimization variables (control points of B-spline curves), the same mechanisms of the classical direct collocation method can be used because the control points with flat outputs are approximately the same sizes.
Using the concept of differential flatness in direct collocation, by reducing the dimensional space of the trajectory optimization problem, the number of variables and optimization constraints of the non-linear programming problem decreases, and the solution speed increases. By using B-spline curves in direct collocation, a continuous approximation of variables is created, and the need to use more time nodes to increase the accuracy of the approximation is eliminated.
This combined method, regardless of the possibility of online trajectory optimization, is also important and valuable. That is, even if it is used as an offline method, it is more desirable and accurate than conventional trajectory optimization methods. This is in contrast to other online trajectory optimization approaches, which are not

interesting to use offline. In the combined method, the accuracy has not been sacrificed for the speed of solving, and both of them are simultaneously improved. In this approach, due to the benefit of the concept of differential flatness, the equation between variables is established with time derivatives and based on an analytical equation, while in the classical direct collocation method, the equation between variables is established by applying discrete equations in consecutive time nodes. As a result, the accuracy of the solution is higher in the combined approach. The solution speed is also much higher than the classical direct collocation method due to the reduction of the number of optimization variables and constraints.
In this paper, we use IPOPT software to solve non-linear programming problems. This software can solve large-scale non-linear programming problems with high accuracy and speed by using the primal-dual interior-point method.

Implementation of the online combined method:

As we know, classical methods of solving trajectory optimization and optimal control problems lead to open-loop solutions. The optimal solutions obtained from these methods are obtained only on the basis of the existing mathematical model and specified and predetermined boundary conditions. Therefore, due to model uncertainties and perturbations, the application of optimal controls and implementation of optimal trajectories in practice will not lead to the satisfaction of boundary conditions and constraints.
In order to achieve a closed-loop solution for trajectory optimization problems, it is necessary to perform the trajectory optimization process online during the mission to take into account the instantaneous conditions of the vehicle and the mission. Due to the instantaneous changes of the state variables, it is necessary to perform the process of optimizing the trajectory online in an instantaneous way, which is not possible. Solving trajectory optimization problems is very time-consuming due to their many complexities. If the time to solve these problems can be reduced as much as possible, the instantaneous trajectory optimization can be achieved to some extent. In this case, the trajectory optimization problem is defined based on the instantaneous conditions of the vehicle and is solved in the shortest time, and control commands are applied. In accordance with the new conditions of the vehicle, the problem of optimizing the next trajectory is defined, and this cycle is followed until the final conditions of the mission are reached. Accordingly, the online trajectory optimization process takes place in the form of a control loop in which the trajectory optimizer acts as a non-linear controller. In this loop, system outputs are fed back to the controller at specified time intervals. Figure 5 shows a block diagram of this control method.

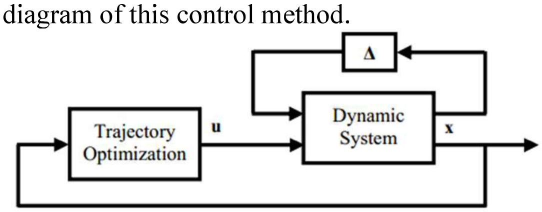

Figure 5- Block diagram of the new method

Online trajectory optimization in the form of a closed-control loop can be considered a new control method in which control is based on trajectory optimization. This control mechanism is similar to what is done in the model predictive control approach. In predictive control, first, an open-loop trajectory is obtained by solving a constrained optimal control problem in a definite and limited horizon of time, and the initial conditions of which are considered in accordance with the instantaneous conditions of the state variables. Then, the calculated optimal controls are applied to the dynamic system in a small part of the mentioned time horizon. By repeating this process, a control loop is created. In fact, this closed-loop control is obtained by calculating the optimal trajectories from the instantaneous conditions of the state variables [15]. Today, this method is more commonly known as receding horizon control. Because in it, optimal control commands are obtained for future time horizons. The term receding horizon control is a more accurate expression of the performance of this control method than the model predictive control. Receding horizon control has a successful function in controlling industrial processes. Of course, this success has been due to the relatively slow dynamics of industrial processes. Receding horizon control algorithms requires a lot of computation and, if implemented improperly, will lead to divergence or poor stability. These problems have led to the avoidance of using this control method in non-linear systems with fast dynamics. Today, with the development and expansion of inexpensive yet powerful computing tools, as well as a better understanding of the stability characteristics of receding horizon control, this control method has been revived. The use of receding horizon control to control aerospace has been suggested and analyzed by some researchers [16]. Using the receding horizon control in the field of aerospace has many advantages. The most important of which is the ability of this control method to consider the state and control constraints. In this method, control commands are determined by considering the constraints and limitations of the problem. Another valuable feature of this method is the possibility of changing the structure of the trajectory optimization problem at any time, which creates extraordinary mission flexibility for aerospace vehicles. In this method, due to solving the trajectory optimization problem in short intervals, it is possible to change the objective functions and constraints each time the problem is solved. That is, each issue can be defined in accordance with the conditions of the vehicle and the mission, with a new structure. For example, in the middle of guiding a missile to intercept a target, we can change the mission and direct the missile to another target. In this method, the dynamic model used to determine the optimal trajectory can be changed for different parts of the mission. Various approaches have been proposed to implement receding horizon control and create control loops based on the methods for solving trajectory optimization problems. Based on these approaches, the effect of instantaneous and final conditions and time horizons of application of control commands is determined. The approach we use in this paper is as follows:

In this approach, after setting up the control loop, the trajectory optimization problem is solved online for a specific and limited time horizon $t_{horizon}$ using the instantaneous state of the state variables. If we assume that the time to calculate the optimal trajectory in this approach is equal to t_{sample}, we apply the optimal control of the calculated trajectory to the system in the next t_{sample}. At the same time (in the second t_{sample}), we calculate a new optimal trajectory based on the instantaneous state of the vehicle and apply the optimal control of the calculated trajectory in the third t_{sample} to the system. By continuing this process until the end of the mission, the vehicle has been able to carry out its mission using the optimally calculated trajectory online. In Figure 6, this approach is shown schematically. It should be noted that by applying the calculated optimal controls, the expected optimal trajectories will not necessarily be traversed because due to various factors such as model uncertainties, perturbations, random factors, and anything that is not accurately modeled in trajectory optimization, the trajectory taken will be different from the expected trajectory. Therefore, when calculating the new optimal trajectory, the expected conditions are not used, and the actual conditions of the vehicle are used. As shown in Figure 6, when applying the optimal control commands, only the part of the calculated control commands that is relevant to the next time period t_{sample} is applied. This causes the computational delays to have the least impact on the control process and the optimal control commands to be applied only at their respective time intervals.

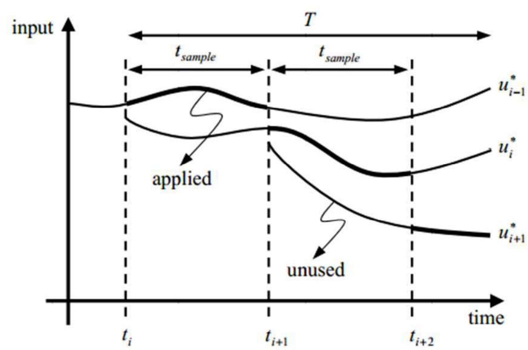

Figure 6- Illustration of a timing scheme

In the implementation of receding horizon control, the value of $t_{horizon}$ is determined in accordance with the dynamic nature of the mission and the vehicle. The value of t_{sample}, is determined based on the time of solving the trajectory optimization problem. This time can be considered as both fixed and variable. In the variable case, whenever the optimal trajectory calculation is completed, the optimal control is applied, and the new optimal trajectory calculation is started based on the instantaneous conditions of the vehicle. Obviously, the value of $t_{horizon}$ must be much higher than t_{sample} in order to implement the optimal trajectory more accurately. Also, the t_{sample} must be small enough for the system time constant to be able to optimize the trajectory based on instantaneous conditions.

Simulation and numerical solution

In this section, we want to present a numerical example of the application of the proposed guidance law and present the results of the relevant simulation. In this simulation, the objective function is to minimize the time (maximize the collision velocity).

In this section, in addition to applying the combined guidance law, we apply the PPN and TPN guidance laws as two examples of optimal classical laws on the problem to allow the comparison of the proposed method with the classical optimal methods.

In order to implement the laws of guidance, we consider the minimum height required to carry out guidance instructions to be 40 km. Because at altitudes above this value, the steering commands are not able to change the direction of movement of the vehicle due to the thin atmosphere. In the simulation process, due to the use of random variables with normal distribution, it is necessary to apply Monte Carlo analysis for an acceptable number of runs. Therefore, in this section, 100 different runs have been performed for each analysis.

For analyzing the solutions, we calculate the three standard deviations (3σ) of range error, the mean final velocity, and the angle of attack Root Mean Square (RMS).

If we consider uncertainties for the initial conditions and some of the model parameters, the vehicle impacts different points. Figure 7 shows the trajectories for 100 runs without applying guidance methods and control commands. For this simulation, 3σ of range error is 12,258 m, and the mean final velocity is 822 m/s.

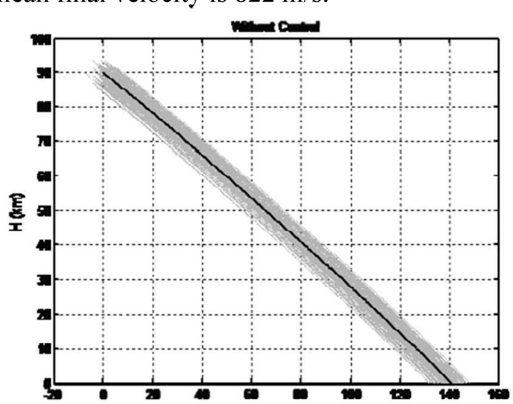

Figure 7- Simulation without control

Table 1 presents the simulation results for 100 runs with the classical PPN and TPN guidance laws and the proposed new method. Also, in Figures 8 to 10, an example of the generated commands by each of the three guidance methods is shown.

Table 1 - Comparison of three methods in the presence of model uncertainties

Method	3σ Range Error (m)	Mean V_f (m/s)	Mean α RMS (deg)
PPN	1.21	763.70	1.71
TPN	1.27	761.63	1.74
New	5.69	952.14	1.36

As can be seen, all three guidance methods have resulted in an acceptable collision error. However, the new guidance law has been able to increase the velocity of the collision significantly, and at the same time, it has been done with less control effort. In the new guidance law, the manner of changing control commands, unlike the other two laws, is smooth and non-fluctuating. PPN and TPN laws, because they are unable to take into account command constraints, lead to saturation in controls. In this case, the output command is more than the amount of control applied to the vehicle, and the vehicle does not fully follow the guidance command due to saturation. While in the new guidance law (combined method), it is possible to define the limitations of control commands, and the guidance law optimizes commands based on these limitations.

Another point is that the optimality of the classical guidance laws is based on very simple kinematic modeling, and the purpose of such guidance laws is simply to reduce the collision error. However, in the new guidance law, the objective function was to minimize the time (maximize collision velocity), the effect of which is clearly visible in Table 1.

The new guidance law works in such a way that the vehicle approaches the target in the shortest time, and the rate of deceleration is minimal to maximize the impact velocity with the target.

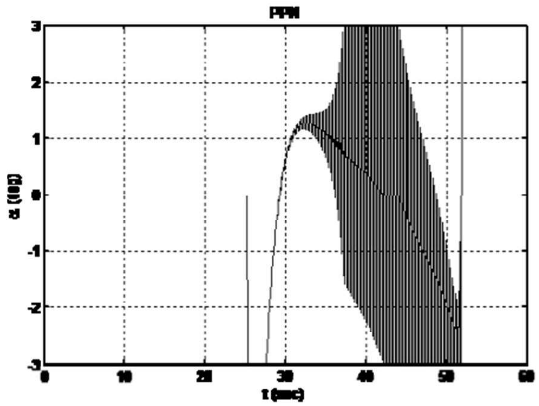

Figure 8 - Generated guidance commands by PPN guidance law

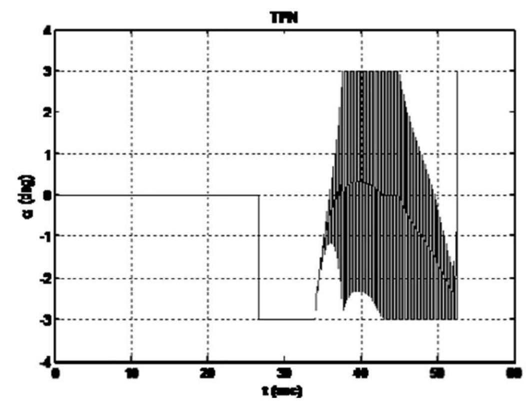

Figure 9 - Generated guidance commands by TPN guidance law

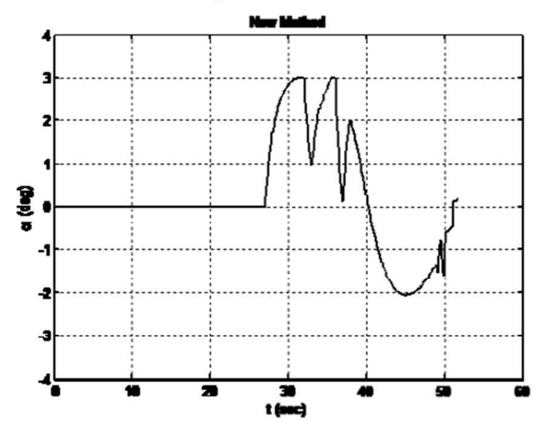

Figure 10 - Generated guidance commands by the new guidance law

Conclusion

A new guidance law based on online trajectory optimization was proposed in the present paper. By using a combined method, this law can generate guidance commands by the instantaneous conditions of the vehicle and the target. This new method can show better performance than conventional guidance laws. It can be used in various applications due to its high flexibility in defining dynamic models, objective functions, and point and path constraints. In the proposed guidance method, the vehicle's best performance can be achieved with a minimum of control effort, and the maximum velocity of hitting the target can be achieved by producing smooth and low-fluctuating guidance commands.

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